Verifying Trig. Functions white-board review

Example

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sec x} = 1$$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\frac{1}{\cos x}}$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos^2 x}{1}$

not an identity

Example

$$\frac{\sin x \csc x}{\cot x} = \tan x$$

$\frac{\sin x \cdot \frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$

$\frac{1}{\frac{\cos x}{\sin x}}$

$\frac{\sin x}{\cos x}$

$\tan x$ ✓

Example

$$\frac{1 - \tan^2 x}{\cot^2 x - 1} = \tan^2 x$$

$\frac{1 - \frac{\sin^2 x}{\cos^2 x}}{\frac{\cos^2 x}{\sin^2 x} - 1} \Rightarrow \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x - \sin^2 x}{\sin^2 x}}$

$\Rightarrow \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \cdot \frac{\sin^2 x}{\cos^2 x - \sin^2 x}$

$\Rightarrow \frac{\sin^2 x}{\cos^2 x} \Rightarrow \tan^2 x$ ✓

Example

$$\csc\theta \sin\theta - \sin^2\theta = \cos^2\theta$$

$\Rightarrow \frac{1}{\sin\theta} \cdot \sin\theta - \sin^2\theta$

$\Rightarrow 1 - \sin^2\theta$

$\Rightarrow \cos^2\theta$ ✓

switch

Example

$$\frac{\tan x - \sin x}{\sin x \tan x} = \frac{\sin x \tan x}{\tan x + \sin x}$$

$\frac{\frac{\sin x}{\cos x} - \sin x}{\sin x \frac{\sin x}{\cos x}} \Rightarrow \frac{\frac{\sin x}{\cos x} - \frac{\sin x \cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}} \Rightarrow \frac{\frac{\sin x - \sin x \cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}}$

$\Rightarrow \frac{\sin x(1-\cos x)}{\cos x} \cdot \frac{\cos x}{\sin^2 x} \Rightarrow \frac{1-\cos x}{\sin x} \cdot \frac{(1+\cos x)}{(1+\cos x)}$

$\Rightarrow \frac{1-\cos^2 x}{\sin(1+\cos x)} \Rightarrow \frac{\sin^2 x}{\sin(1+\cos x)} =$

$\Rightarrow \frac{\sin x}{1+\cos x}\left(\frac{\tan x}{\tan x}\right) \Rightarrow \frac{\sin x \tan x}{\tan x + \cos x\left(\frac{\sin x}{\cos x}\right)}$

$\Rightarrow \frac{\sin x \tan x}{\tan x + \sin x}$ ✓

Example

$\sin x\,(\cot x + \tan x) = \sec x$

$\Rightarrow \sin x\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$

$\Rightarrow \cos x + \frac{\sin^2 x}{\cos x} \Rightarrow \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} \Rightarrow \frac{\cos^2 x + \sin^2 x}{\cos x}$

$\Rightarrow \frac{1}{\cos x} \Rightarrow \sec x$ ✓

Example

$(\csc x + \cot x)(\csc x - \cot x) = 1$

$\csc^2 x - \cot^2 x = 1$

$\therefore 1 = 1$ ✓

OR $\csc^2 x - \cot^2 x$

$\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} \Rightarrow \frac{1-\cos^2 x}{\sin^2 x}$

$\Rightarrow \frac{\sin^2 x}{\sin^2 x}$

$\Rightarrow 1$ ✓

Example

$\csc x + \cot x = \frac{\sin x}{1 - \cos x}$

$\Rightarrow \frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$\Rightarrow \frac{1+\cos x}{\sin x}\left(\frac{\sin x}{\sin x}\right)$

$\Rightarrow \frac{(1+\cos x)\sin x}{\sin^2 x}$

$\Rightarrow \frac{(1+\cos x)\sin x}{1-\cos^2 x}$ ← Diff. of ☐

$\Rightarrow \frac{(1+\cos x)\sin x}{(1+\cos x)(1-\cos x)} \Rightarrow \frac{\sin x}{1-\cos x}$

Example

$8\csc^2 x - 3\cot^2 x = 3 + 5\csc^2 x$

$\Rightarrow \frac{8}{\sin^2 x} - \frac{3\cos^2 x}{\sin^2 x}$

$\Rightarrow \frac{8 - 3\cos^2 x}{\sin^2 x}$

$\Rightarrow \frac{8 - 3(1 - \sin^2 x)}{\sin^2 x}$

$\Rightarrow \frac{8 - 3 + 3\sin^2 x}{\sin^2 x}$

$\Rightarrow \frac{5 + 3\sin^2 x}{\sin^2 x}$

$\Rightarrow \frac{5}{\sin^2 x} + \frac{3\sin^2 x}{\sin^2 x} \Rightarrow 3 + 5\csc^2 x$

Fix

Example

$\frac{1}{1-\sec x} + \frac{1}{1+\sec x} = -2\cot^2 x$

$(1+\sec x)$ $(1-\sec x)$

$\Rightarrow \frac{1+\sec x + 1 - \sec x}{(1-\sec x)(1+\sec x)}$

$\Rightarrow \frac{2}{1-\sec^2 x} \Rightarrow \frac{2}{1-\frac{1}{\cos^2 x}}$

$\Rightarrow \frac{2}{\frac{\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x}} \Rightarrow \frac{2}{\frac{\cos^2 x - 1}{\cos^2 x}} \Rightarrow \frac{2\cos^2 x}{\cos^2 x - 1}$

$\Rightarrow \frac{2\cos^2 x}{-\sin^2 x} \Rightarrow -2\cot^2 x$

Example

$\frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$

$\frac{\frac{1}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$

$\Rightarrow \frac{\frac{1+\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$

$\Rightarrow \frac{1+\cos x}{\cos x} \cdot \frac{\cos x}{\sin x}$

$\Rightarrow \frac{1+\cos x}{\sin x}\left(\frac{1-\cos x}{1-\cos x}\right)$

$\Rightarrow \frac{1-\cos^2 x}{\sin x(1-\cos x)}$

$\Rightarrow \frac{\sin^2 x}{\sin x(1-\cos x)}$

$\Rightarrow \frac{\sin x}{1-\cos x}$ ✓

Example

$$\frac{\tan x}{\sec x - 1} = \frac{\sec x + 1}{\tan x}$$

$$\Rightarrow \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} - 1\left(\frac{\cos x}{\cos x}\right)}$$

$$\Rightarrow \frac{\frac{\sin x}{\cos x}}{\frac{1-\cos x}{\cos x}}$$

$$\Rightarrow \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1-\cos x}$$

$$\Rightarrow \frac{\sin x}{1-\cos x}\,\frac{(1+\cos x)}{(1+\cos x)}$$

$$\Rightarrow \frac{\sin x(1+\cos x)}{1-\cos^2 x}$$

$$\Rightarrow \frac{\sin x(1+\cos x)}{\sin^2 x}$$

$$\Rightarrow \frac{1+\cos x}{\sin x}\left(\frac{\sin x}{\cos x}\right)$$

$$\frac{1+\cos x}{\cos x}$$

$$\left(\frac{1}{\cos x} + \frac{\cos x}{\cos x}\right)$$

$$(\sec x + 1)\left(\frac{1}{\tan x}\right)$$

$$\frac{\sec x + 1}{\tan x}$$ ✓

$$\frac{\frac{1}{\cos x} + 1}{\frac{\sin x}{\cos x}}$$

$$\frac{1+\cos x}{\cos x} \cdot \frac{\cos x}{\sin x}$$

$$\frac{1+\cos x}{\sin x}$$

Example

$$\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$$

$$\Rightarrow \frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x}{1}$$

$$\Rightarrow \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$$

$$\Rightarrow \frac{\cos^2 x(1-\sin^2 x)}{\sin^2 x}$$

$$\Rightarrow \frac{\cos^2 x(\cos^2 x)}{\sin^2 x}$$

$$\Rightarrow \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\cos^2 x}{\sin^2 x}$$

$$\Rightarrow \cot^2 x \cdot \frac{\cos^2 x}{\sin^2 x} \cdot \left(\frac{\sin^2 x}{1}\right)$$

$$\Rightarrow \cot^2 x \cos^2 x$$ ✓