Pre-Calculus Honors Name Key

Section 5.4-5.5 Identities with Trig Review

Use the angle sum or difference identity to find the exact value of each.

1) $\cos 255° = \cos(45+210)$

$\cos 45 \cos 210 - \sin 45 \sin 210$

$\left(\frac{\sqrt2}{2}\right)\left(-\frac{\sqrt3}{2}\right) - \left(\frac{\sqrt2}{2}\right)\left(\frac{-1}{2}\right)$

$-\frac{\sqrt6}{4} + \frac{\sqrt2}{4} = \boxed{\frac{\sqrt2 - \sqrt6}{4}}$

2) $\sin 15° = \sin(45-30)$

$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{\sqrt2}{2}\left(\frac{1}{2}\right) - \left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt3}{2}\right) = \frac{\sqrt2}{4} - \frac{\sqrt6}{4} = \boxed{\frac{\sqrt2-\sqrt6}{4}}$

3) $\cos \frac{13\pi}{12} = \cos\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$\cos\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$

$-\frac{\sqrt2}{2}\left(\frac{1}{2}\right) - \frac{\sqrt2}{2}\left(\frac{\sqrt3}{2}\right)$

$-\frac{\sqrt2}{4} - \frac{\sqrt6}{4} = \boxed{\frac{-\sqrt2-\sqrt6}{4}}$

4) $\tan -\frac{\pi}{12} = \tan\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$\frac{\tan \pi/4 - \tan \pi/3}{1 + \tan \pi/4 \tan \pi/3} = \boxed{\frac{1-\sqrt3}{1+\sqrt3} \text{ or } \sqrt3 - 2}$

5) $\tan \frac{7\pi}{12} = \tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$\frac{\tan \pi/4 + \tan \pi/3}{1 - \tan \pi/4 \tan \pi/3} = \boxed{\frac{1+\sqrt3}{1-\sqrt3} \text{ or } -\frac{4+\sqrt3}{2}}$

6) $\sin \frac{13\pi}{12}$ $\sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$\sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$

$\frac{\sqrt2}{2}\left(\frac{1}{2}\right) + \left(-\frac{\sqrt2}{2}\right)\left(\frac{\sqrt3}{2}\right)$

$\frac{\sqrt2}{4} + -\frac{\sqrt6}{4} = \boxed{\frac{\sqrt2-\sqrt6}{4}}$

Use a double-angle identity to find the exact value of each expression.

7) $\tan \theta = -\frac{5}{4}$ and $\frac{3\pi}{2} < \theta < 2\pi$

$5^2 + 4^2 = c^2$

$\sqrt{41} = c$

$\sin\theta = \frac{-5}{\sqrt{41}}$

$\cos\theta = \frac{4}{\sqrt{41}}$

Find $\tan 2\theta$, $\sin 2\theta$, $\cos 2\theta$

$\tan 2\theta = \frac{2\left(-\frac{5}{4}\right)}{1 - \left(-\frac{5}{4}\right)^2} = -\frac{5}{2} \div \left(\frac{16-25}{16}\right) = -\frac{5}{2} \cdot \frac{16}{-9} = \boxed{\frac{40}{9}}$

$\sin 2\theta = 2\left(-\frac{5}{\sqrt{41}}\right)\left(\frac{4}{\sqrt{41}}\right) = \boxed{-\frac{40}{41}}$

$\cos 2\theta = \left(\frac{4}{\sqrt{41}}\right)^2 - \left(\frac{-5}{\sqrt{41}}\right)^2 = \frac{16}{41} - \frac{25}{41} = \boxed{\frac{-9}{41}}$

8) $\cot \theta = \frac{4}{3}$ and $\pi < \theta < \frac{3\pi}{2}$

$\tan\theta = \frac{3}{4}$, $\sin\theta = \frac{-3}{5}$

$\cos\theta = -\frac{4}{5}$

Find $\tan 2\theta$, $\sin 2\theta$, $\cos 2\theta$

$\tan 2\theta = \frac{2(3/4)}{1 - (3/4)^2} = \frac{3/2}{\frac{16-9}{16}} = \frac{3}{2} \cdot \frac{16}{7} = \boxed{\frac{24}{7}}$

$\sin 2\theta = 2\left(\frac{-3}{5}\right)\left(-\frac{4}{5}\right) = \boxed{\frac{24}{25}}$

$\cos 2\theta = \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

Solve each equation for $0 \le \theta \le 2\pi$.

9) $\cos\left(\theta + \frac{\pi}{4}\right) - \cos\left(\theta - \frac{\pi}{4}\right) = 1$

$\cos\theta\cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4} - \left[\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}\right] = 1$

$\cancel{\cos\theta\left(\frac{\sqrt2}{2}\right)} - \sin\theta\left(\frac{\sqrt2}{2}\right) - \cancel{\cos\theta\left(\frac{\sqrt2}{2}\right)} - \sin\theta\left(\frac{\sqrt2}{2}\right) = 1$

$-\cancel{2}\sin\theta\left(\frac{\sqrt2}{\cancel{2}}\right) = 1$

$-\sin\theta(\sqrt2) = 1$

$\sin\theta = \frac{-1}{\sqrt2} = -\frac{\sqrt2}{2}$

$\boxed{\theta = \frac{4\pi}{3}, \frac{5\pi}{3}}$

10) $-1 = 5 - 3\sec -2\theta$

$-5 \quad -5$

$-6 = -3\sec(-2\theta)$

$2 = \sec(-2\theta)$

$2 = \frac{1}{\cos(-2\theta)}$

$2\cos(-2\theta) = 1$

$\cos(-2\theta) = \frac{1}{2}$

$-2\theta = \frac{\pi}{3}$, $-2\theta = \frac{5\pi}{3}$

$\boxed{\theta = -\frac{\pi}{6}, \ \theta = -\frac{5\pi}{6}}$

11) $2\cos \theta + \sin(2\theta) = 0$

$2\cos\theta + 2\sin\theta\cos\theta = 0$

$2\cos\theta(1 + \sin\theta) = 0$

$2\cos\theta = 0 \qquad 1 + \sin\theta = 0$

$\cos\theta = 0 \qquad \sin\theta = -1$

$\theta = \pi/2, \frac{3\pi}{2} \qquad \theta = 3\pi/2$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

12) $\cos \theta = \sin(2\theta)\sin \theta$

$\cos\theta = 2\sin\theta\cos\theta\sin\theta$

$\cos\theta = 2\sin^2\theta\cos\theta$

$\cos\theta - 2\sin^2\theta\cos\theta = 0$

$\cos\theta(1 - 2\sin^2\theta) = 0$

$\cos\theta = 0$ or $1 - 2\sin^2\theta = 0$

$\sin\theta = -\frac{\sqrt2}{2}$

$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \theta = \frac{5\pi}{4}, \frac{7\pi}{4}}$

Verify the identity.

13) $\sin\left(\theta + \frac{\pi}{2}\right) + \cos(\cancel{-\theta + \pi}) = 0$ $\pi - \theta$

$\sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2} + \cos\pi\cos\theta + \sin\pi\sin\theta$

$\sin\theta(0) + \cos\theta(1) + \cos\theta(-1) + \sin\theta(0)$

$\cos\theta - \cos\theta$

$0 = 0$

* 14) $\sin(3\theta) = 3\sin \theta - 4\sin^3 \theta$

hint: use 3θ as $2\theta + \theta$

$\sin(2\theta + \theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= \sin 2\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta \rightarrow$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$ ✓