

Section 5.4-5.5 Identities with Trig Review

Use the angle sum or difference identity to find the exact value of each.

1) $\cos 255^\circ = \cos(45+210)$

$$\begin{aligned} & \cos 45 \cos 210 - \sin 45 \sin 210 \\ & \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right) \\ & \frac{-\sqrt{6} + \sqrt{2}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}} \end{aligned}$$

3) $\cos \frac{13\pi}{12} = \cos\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$$\begin{aligned} & \cos \frac{3\pi}{4} \cos \frac{\pi}{3} - \sin \frac{3\pi}{4} \sin \frac{\pi}{3} \\ & -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) \\ & \frac{-\sqrt{2} - \sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}} \end{aligned}$$

5) $\tan \frac{7\pi}{12} = \tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$$\frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{3}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{3}} = \boxed{\frac{1+\sqrt{3}}{1-\sqrt{3}} \text{ or } \frac{-4+\sqrt{3}}{2}}$$

2) $\sin 15^\circ = \sin(45-30)$

$$\begin{aligned} & \sin 45 \cos 30 - \cos 45 \sin 30 \\ & \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}} \end{aligned}$$

4) $\tan -\frac{\pi}{12} = \tan\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

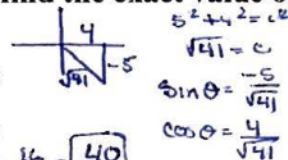
$$\frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{3}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{3}} = \boxed{\frac{1-\sqrt{3}}{1+\sqrt{3}} \text{ or } \sqrt{3}-2}$$

6) $\sin \frac{13\pi}{12} = \sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$$\begin{aligned} & \sin \frac{3\pi}{4} \cos \frac{\pi}{3} + \cos \frac{3\pi}{4} \sin \frac{\pi}{3} \\ & \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ & \frac{\sqrt{2} - \sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}} \end{aligned}$$

Use a double-angle identity to find the exact value of each expression.

7) $\tan \theta = -\frac{5}{4}$ and $\frac{3\pi}{2} < \theta < 2\pi$



$$\begin{aligned} & 5^2 + 4^2 = c^2 \\ & \sqrt{41} = c \\ & \sin \theta = \frac{-5}{\sqrt{41}} \\ & \cos \theta = \frac{4}{\sqrt{41}} \end{aligned}$$

Find $\tan 2\theta$, $\sin 2\theta$, $\cos 2\theta$

$$\tan 2\theta = \frac{2\left(-\frac{5}{4}\right)}{1 - \left(-\frac{5}{4}\right)^2} = \frac{-\frac{5}{2}}{1 - \frac{25}{16}} = \frac{-\frac{5}{2}}{\frac{16-25}{16}} = \frac{-\frac{5}{2} \cdot 16}{-9} = \boxed{\frac{40}{9}}$$

$$\sin 2\theta = 2\left(-\frac{5}{\sqrt{41}}\right)\left(\frac{4}{\sqrt{41}}\right) = \boxed{\frac{-40}{41}}$$

$$\cos 2\theta = \left(\frac{4}{\sqrt{41}}\right)^2 - \left(\frac{-5}{\sqrt{41}}\right)^2 = \frac{16}{41} - \frac{25}{41} = \boxed{\frac{-9}{41}}$$

Solve each equation for $0 \leq \theta \leq 2\pi$.

9) $\cos\left(\theta + \frac{\pi}{4}\right) - \cos\left(\theta - \frac{\pi}{4}\right) = 1$

$$\cos \theta \cos \frac{\pi}{4} - \sin \theta \sin \frac{\pi}{4} - \left[\cos \theta \cos \frac{\pi}{4} + \sin \theta \sin \frac{\pi}{4} \right] = 1$$

$$\begin{aligned} & \cos \theta \left(\frac{\sqrt{2}}{2}\right) - \sin \theta \left(\frac{\sqrt{2}}{2}\right) - \cos \theta \left(\frac{\sqrt{2}}{2}\right) - \sin \theta \left(\frac{\sqrt{2}}{2}\right) = 1 \\ & -2\sin \theta \left(\frac{\sqrt{2}}{2}\right) = 1 \\ & -\sin \theta (\sqrt{2}) = 1 \end{aligned}$$

$$\sin \theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$$

11) $2\cos \theta + \sin(2\theta) = 0$

$$2\cos \theta + 2\sin \theta \cos \theta = 0$$

$$2\cos \theta (1 + \sin \theta) = 0$$

$$2\cos \theta = 0 \quad 1 + \sin \theta = 0$$

$$\cos \theta = 0 \quad \sin \theta = -1$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \theta = \frac{3\pi}{2}$$

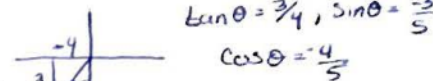
Verify the identity.

13) $\sin\left(\theta + \frac{\pi}{2}\right) + \cos(-\theta + \pi) = 0$

$$\begin{aligned} & \sin \theta \cos \frac{\pi}{2} + \cos \theta \sin \frac{\pi}{2} + \cos \pi \cos \theta + \sin \pi \sin \theta \\ & \sin \theta (0) + \cos \theta (1) + \cos \theta (-1) + \sin \theta (0) = 0 \end{aligned}$$

$$\cos \theta - \cos \theta = 0$$

8) $\cot \theta = \frac{4}{3}$ and $\pi < \theta < \frac{3\pi}{2}$



$$\begin{aligned} & \tan \theta = \frac{3}{4}, \sin \theta = \frac{-3}{5} \\ & \cos \theta = \frac{-4}{5} \end{aligned}$$

Find $\tan 2\theta$, $\sin 2\theta$, $\cos 2\theta$

$$\tan 2\theta = \frac{2\left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1 - \frac{9}{16}} = \frac{\frac{3}{2}}{\frac{16-9}{16}} = \frac{\frac{3}{2} \cdot 16}{7} = \boxed{\frac{24}{7}}$$

$$\sin 2\theta = 2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right) = \boxed{\frac{24}{25}}$$

$$\cos 2\theta = \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$$

10) $-1 = 5 - 3\sec -2\theta$

$$-6 = -3\sec(-2\theta)$$

$$2 = \sec(-2\theta)$$

$$\sec 2\theta = 2 = \frac{1}{\cos(-2\theta)}$$

$$2\cos(-2\theta) = 1$$

$$\cos(-2\theta) = \frac{1}{2}$$

$$-2\theta = \frac{\pi}{3}, -2\theta = \frac{5\pi}{3}$$

$$\theta = -\frac{\pi}{6}, \theta = -\frac{5\pi}{6}$$

12) $\cos \theta = \sin(2\theta)\sin \theta$

$$\cos \theta = 2\sin \theta \cos \theta \sin \theta$$

$$\cos \theta = 2\sin^2 \theta \cos \theta$$

$$\cos \theta - 2\sin^2 \theta \cos \theta = 0$$

$$\cos \theta (1 - 2\sin^2 \theta) = 0$$

$$\cos \theta = 0 \text{ or } 1 - 2\sin^2 \theta = 0$$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\begin{aligned} & \theta = \frac{\pi}{2}, \frac{3\pi}{2} \\ & \theta = \frac{5\pi}{4}, \frac{7\pi}{4} \end{aligned}$$

* 14) $\sin(3\theta) = 3\sin \theta - 4\sin^3 \theta$

hint: use 3θ as $2\theta + \theta$

$$\begin{aligned} & \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \\ & = \sin 2\theta \cos \theta + (1 - 2\sin^2 \theta) \sin \theta \\ & = 2\sin \theta \cos \theta \cos \theta + \sin \theta - 2\sin^3 \theta \\ & = 2\sin \theta \cos^2 \theta + \sin \theta - 2\sin^3 \theta \end{aligned}$$

$$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$$

$$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$$

$$= 3\sin\theta - 4\sin^3\theta \quad \checkmark$$

