

Sec 5.1 & 5.2 Review

Trig Identities worksheet name: _____

Prove each identity:

1. $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$$\frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x}$$

$$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$\frac{1 - \sin^2 x}{\cos x}$$

$$\Rightarrow \frac{\cos^2 x}{\cos x}$$

$$\Rightarrow \cos x \Rightarrow \frac{1}{\sec x} \checkmark$$

2. $\frac{1 + \cos x}{\sin x} = \csc x + \cot x$

$$\Rightarrow \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$\Rightarrow \csc x + \cot x \checkmark$$

3. $\frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$

$$\Rightarrow \frac{\frac{1}{\cos \theta} \cdot \sin \theta}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} \Rightarrow \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}}$$

$$\Rightarrow \frac{\frac{\sin \theta}{\cos \theta}}{\frac{1}{\sin \theta \cos \theta}} \Rightarrow \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta \cos \theta}{1} \Rightarrow \sin^2 \theta \checkmark$$

4. $\frac{\sec \theta \cdot \tan \theta}{\cos \theta} - \cot \theta = 1$

$$\frac{\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta}}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \Rightarrow \frac{1}{\cos^2 \theta} \cdot \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}$$

$$\Rightarrow \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \Rightarrow \frac{1 - \sin^2 \theta}{\cos^2 \theta}$$

$$\Rightarrow \frac{\cos^2 \theta}{\cos^2 \theta} = 1 \checkmark$$

5. $\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$

$$(1 - \sin^2 y) - \sin^2 y$$

$$1 - 2\sin^2 y \checkmark$$

6. $\csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$

$$\frac{1}{\sin^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} - 1$$

$$\frac{1}{\cos^2 \theta} - \frac{\cos^2 \theta}{\cos^2 \theta}$$

$$\frac{1 - \cos^2 \theta}{\cos^2 \theta} \Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta \checkmark$$

7. $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$

$$\frac{\frac{1}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta} - 1} \Rightarrow \frac{\frac{1}{\cos^2 \theta}}{\frac{1 - \cos^2 \theta}{\cos^2 \theta}} \Rightarrow \frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{1 - \cos^2 \theta}$$

$$\Rightarrow \frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} = \csc^2 \theta \checkmark$$

8. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x$$

$$\Rightarrow \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x}$$

$$\Rightarrow \tan^2 x - \sin^2 x \checkmark$$

9. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$$\begin{aligned} &\Rightarrow \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta \\ &\Rightarrow 2\sin^2\theta + 2\cos^2\theta \\ &\Rightarrow 2(\sin^2\theta + \cos^2\theta) \\ &\Rightarrow 2(1) \\ &\Rightarrow 2 \end{aligned}$$

10. $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$

$$\begin{aligned} &\Rightarrow \sin\theta\tan\theta + \sin\theta\cot\theta + \cos\theta\tan\theta + \cos\theta\cot\theta \\ &\Rightarrow \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right) + \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right) \\ &\Rightarrow \frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1} + \frac{\sin\theta}{1} + \frac{\cos^2\theta}{\sin\theta} \\ &\Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\cos\theta} + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta} \\ &\Rightarrow \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \Rightarrow \sec\theta + \csc\theta \checkmark \end{aligned}$$

11. $\frac{\tan\theta - 1}{\tan\theta + 1} = \frac{1 - \cot\theta}{1 + \cot\theta}$

$$\begin{aligned} &\frac{\frac{\sin\theta}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta} + 1} \Rightarrow \frac{\frac{\sin\theta - \cos\theta}{\cos\theta}}{\frac{\sin\theta + \cos\theta}{\cos\theta}} \Rightarrow \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \cdot \frac{\cos\theta}{\cos\theta} \\ &\Rightarrow \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \left(\frac{1}{\sin\theta}\right) \Rightarrow \frac{\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{1 - \cot\theta}{1 + \cot\theta} \checkmark \end{aligned}$$

12. $\frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$

$$\begin{aligned} &\frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}} \Rightarrow \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} \cdot \frac{\cos^2\theta}{1} \\ &\Rightarrow \cos^2\theta - \sin^2\theta \\ &\Rightarrow 1 - \sin^2\theta - \sin^2\theta \\ &\Rightarrow 1 - 2\sin^2\theta \checkmark \end{aligned}$$

13. $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$

$$\begin{aligned} &\frac{\cos x + 1}{\sin^3 x \cdot \sin x} \Rightarrow \frac{\cos x + 1}{(1 - \cos^2 x)\sin x} \Rightarrow \frac{\cos x + 1}{(1 + \cos x)(1 - \cos x)\sin x} \\ &\Rightarrow \frac{1}{(1 - \cos x)\sin x} \Rightarrow \frac{1}{\sin x} \Rightarrow \frac{\csc x}{1 - \cos x} \checkmark \end{aligned}$$

14. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

$$\begin{aligned} &(\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) \\ &\left(\frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}\right)\left(\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}\right) \\ &\left(\frac{1 + \cos^2 x}{\sin^2 x}\right)\left(\frac{1 - \cos^2 x}{\sin^2 x}\right) \\ &\left(\frac{1 + \cos^2 x}{\sin^2 x}\right)\left(\frac{\sin^2 x}{\sin^2 x}\right) \\ &\left(\frac{1 + \cos^2 x}{\sin^2 x}\right)(1) \Rightarrow \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \Rightarrow \csc^2 x + \cot^2 x \checkmark \end{aligned}$$

15. $\frac{\tan\theta + \cot\theta}{\sec\theta \csc\theta} = \sin\theta + \cos\theta$

$$\begin{aligned} &\frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}} \Rightarrow \frac{\frac{\sin\theta \cdot \cos\theta}{\cos\theta} + \frac{\cos\theta \cdot \sin\theta}{\sin\theta}}{\frac{1}{\sin\theta\cos\theta}} \\ &\Rightarrow \sin\theta + \cos\theta \checkmark \end{aligned}$$

16. $\frac{\sin y + \tan y}{1 + \sec y} = \sin y$

$$\begin{aligned} &\Rightarrow \frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}} \Rightarrow \frac{\frac{\sin y \cos y + \sin y}{\cos y}}{\frac{\cos y + 1}{\cos y}} \\ &\Rightarrow \frac{\sin y (\cos y + 1)}{\cos y} \cdot \frac{\cos y}{\cos y + 1} \\ &\Rightarrow \sin y \checkmark \end{aligned}$$

Part 2: Verifying Trigonometric Identities

17) $\cos x + \sin x \tan x = \sec x$

$$\begin{aligned} & \cos x + \sin x \left(\frac{\sin x}{\cos x} \right) \\ & \left(\frac{\cos x}{\cos x} \right) \cdot \cos x + \frac{\sin^2 x}{\cos x} \\ & \frac{\cos^2 x + \sin^2 x}{\cos x} \\ & \frac{1}{\cos x} \\ & \sec x \quad \checkmark \end{aligned}$$

19) $\frac{1}{\tan x} + \tan x = \frac{\sec^2 x}{\tan x}$

$$\begin{aligned} & \frac{1}{\tan x} + \frac{\tan^2 x}{\tan x} \\ & \frac{1 + \tan^2 x}{\tan x} \\ & \frac{\sec^2 x}{\tan x} \quad \checkmark \end{aligned}$$

21) $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

$$\begin{aligned} & \frac{\cos x}{1 - \sin x} \cdot \frac{(1 + \sin x)}{(1 + \sin x)} \\ & \frac{\cos x (1 + \sin x)}{1 - \sin^2 x} \\ & \frac{\cos x (1 + \sin x)}{\cos^2 x} \\ & \frac{1 + \sin x}{\cos x} \Rightarrow \frac{1}{\cos x} + \frac{\sin x}{\cos x} \\ & = \sec x + \tan x \quad \checkmark \end{aligned}$$

23) $\frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x} = 0$

$$\begin{aligned} & \left(\frac{\sin x}{\sin x} \right) \frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x} \left(\frac{\cos x + 1}{\cos x + 1} \right) \\ & \frac{\sin^2 x + \cos^2 x - 1}{\sin x (\cos x + 1)} \\ & = \frac{1 - 1}{\sin x (\cos x + 1)} \\ & = \frac{0}{\sin x (\cos x + 1)} \\ & = 0 \quad \checkmark \end{aligned}$$

18) $\frac{\csc x - \sin x}{\sin x \csc x} = \csc x - \sin x$

$$\begin{aligned} & \frac{\csc x}{\sin x \csc x} - \frac{\sin x}{\sin x \csc x} \\ & \frac{1}{\sin x} - \frac{1}{\csc x} \\ & \csc x - \sin x \quad \checkmark \end{aligned}$$

20) $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 2 \sec x$

$$\begin{aligned} & \left(\frac{1 + \sin x}{1 + \sin x} \right) \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} \left(\frac{\cos x}{\cos x} \right) \\ & \frac{(1 + \sin x)^2 + \cos^2 x}{\cos x (1 + \sin x)} \\ & \Rightarrow \frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x (1 + \sin x)} \Rightarrow \frac{1 + 2\sin x + 1}{\cos x (1 + \sin x)} \\ & \Rightarrow \frac{2 + 2\sin x}{\cos x (1 + \sin x)} \Rightarrow \frac{2(1 + \sin x)}{\cos x (1 + \sin x)} \Rightarrow \frac{2}{\cos x} \Rightarrow 2 \sec x \quad \checkmark \end{aligned}$$

22) $\frac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \cos^2 x$

$$\begin{aligned} & \frac{\cos^2 x - \sin^2 x}{1 - \frac{\sin^2 x}{\cos^2 x}} \\ & \left(\frac{\cos x}{\cos x} \right) \frac{\cos^2 x - \sin^2 x}{1 - \frac{\sin^2 x}{\cos^2 x}} \\ & \Rightarrow \frac{\cos^2 x - \sin^2 x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} \\ & \Rightarrow \cos^2 x - \sin^2 x \cdot \frac{\cos^2 x}{\cos^2 x - \sin^2 x} = \cos^2 x \quad \checkmark \end{aligned}$$

24) $\frac{\sin^2 x + \cos^2 x + \cot^2 x}{1 + \tan^2 x} = \cot^2 x$

$$\begin{aligned} & \Rightarrow \frac{1 + \cot^2 x}{1 + \tan^2 x} \\ & = \frac{\csc^2 x}{\sec^2 x} \\ & = \frac{1}{\sin^2 x} \cdot \frac{1}{\cos^2 x} \\ & \Rightarrow \frac{1}{\sin^2 x} \cdot \frac{\cos^2 x}{1} \Rightarrow \frac{\cos^2 x}{\sin^2 x} = \cot^2 x \quad \checkmark \end{aligned}$$

