TRIGONOMETRY II - LESSON TWO

Name: _____

PART VI CONJUGATES

Sometimes you will get identities that can't be broken down any further. In these cases, you can multiply numerator & denominator by the conjugate. This will convert the fraction into something that will give you identities to work with.

The conjugate is obtained by taking a binomial from the original expression and changing the sign in the middle.

Example 1: Prove $\frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x}$

$\frac{1+\cos x}{\sin x}$ has the conjugate: **1 - cos x**

$$\frac{1+\cos x}{\sin x}$$

$$= \frac{1+\cos x}{\sin x}\left(\frac{1-\cos x}{1-\cos x}\right)$$

$$= \frac{1-\cos^2 x}{\sin x(1-\cos x)}$$

$$= \frac{\sin^2 x}{\sin x(1-\cos x)}$$

$$= \frac{\sin x}{1-\cos x}$$

Prove each of the following identities:

1) $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$

$= \frac{\cos x}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right)$

$= \frac{\cos x(1+\sin x)}{1-\sin^2 x}$

$= \frac{\cos x(1+\sin x)}{\cos^2 x}$

$= \frac{1+\sin x}{\cos x}$ ✓

2) $\frac{1}{1-\sin x} = \frac{1+\sin x}{\cos^2 x}$

$= \frac{1}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right)$

$= \frac{1+\sin x}{1-\sin^2 x}$

$= \frac{1+\sin x}{\cos^2 x}$ ✓

3) $\frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$

$\frac{1-\cos x}{\sin x}\left(\frac{1+\cos x}{1+\cos x}\right)$

$\frac{1-\cos^2 x}{\sin x(1+\cos x)}$

$\frac{\sin^2 x}{\sin x(1+\cos x)}$

$\frac{\sin x}{1+\cos x}$ ✓

4) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

$\frac{1-\sin x}{\cos x}\left(\frac{1+\sin x}{1+\sin x}\right)$

$\frac{1-\sin^2 x}{\cos x(1+\sin x)}$

$\frac{\cos^2 x}{\cos x(1+\sin x)}$

$\frac{\cos x}{1+\sin x}$ ✓

5. $\sin\left(\frac{\pi}{2}-x\right)\csc x = \cot x$

$\cos x\left(\frac{1}{\sin x}\right)$

$\frac{\cos x}{\sin x} \Rightarrow \cot x$ ✓

6. $\cot\left(\frac{\pi}{2}-x\right)\cos x = \sin x$

$\tan x(\cos x)$

$\frac{\sin x}{\cos x}(\cos x)$

$\sin x$ ✓

7. $\frac{\cot(-x)}{\csc x} = -\cos x$

$\Rightarrow \frac{-\cot x}{\csc x}$

$\Rightarrow \frac{-\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} \Rightarrow -\frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} = -\cos x$ ✓

8. $\frac{\csc\left(\frac{\pi}{2}-x\right)}{\tan(-x)} = -\csc x$

$\Rightarrow \frac{\sec x}{-\tan\theta}$

$\Rightarrow \frac{\frac{1}{\cos x}}{-\frac{\sin x}{\cos x}} \Rightarrow \frac{1}{\cos x} \cdot \frac{\cos x}{-\sin x} = \frac{1}{-\sin x} = -\csc x$ ✓

9. $\sin(-x) + \cos(-x)\cot(-x) = -\csc x$

$-\sin x + \cos x(-\cot x)$

$-\sin x + \cos x\left(-\frac{\cos x}{\sin x}\right)$

$\left(\frac{\sin x}{\sin x}\right) -\frac{\sin x}{1} + \frac{\cos^2 x}{\sin x} \Rightarrow \frac{-\sin^2 x - \cos^2 x}{\sin x} = -\frac{(\sin^2 x + \cos^2 x)}{\sin x} \Rightarrow -\frac{1}{\sin x}$

$\Rightarrow -\csc x$ ✓

10. $\cos(-x)\csc(-x) - \cot(-x) = 0$

$\cos x(-\csc x) + \cot x$

$\cos x\left(-\frac{1}{\sin x}\right) + \cot x$

$-\frac{\cos x}{\sin x} + \cot x$

$-\cot x + \cot x$

0 ✓