

# TRIGONOMETRY II - LESSON TWO

PART V

OTHER PROOFS

$$1) \frac{3 \tan x}{1 + \tan^2 x} = 3 \sin x \cos x$$

$$= \frac{3 \tan x}{\sec^2 x} \Rightarrow \frac{3 \sin x}{\cos x} \cdot \frac{1}{\cos^2 x} \Rightarrow \frac{3 \sin x}{\cancel{\cos x}} \cdot \frac{\cos^2 x}{1} = 3 \sin x \cos x \checkmark$$

$$2) \frac{1}{1 + \cot^2 x} = \sin^2 x$$

$$\frac{1}{\csc^2 x} = \frac{1}{\frac{1}{\sin^2 x}} = 1 \cdot \frac{\sin^2 x}{1} = \sin^2 x \checkmark$$

$$3) \sec^2 x - \cos^2 x - \sin^2 x = \tan^2 x$$

$$= \sec^2 x - (\cos^2 x + \sin^2 x)$$

$$\Rightarrow \sec^2 x - 1$$

$$\Rightarrow \tan^2 x$$

$$4) (\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$$

$$= \sin^2 x + 2 \sin x \cos x + \cos^2 x + \sin^2 x - 2 \sin x \cos x + \cos^2 x$$

$$= 2 \sin^2 x + 2 \cos^2 x$$

$$= 2 (\sin^2 x + \cos^2 x)$$

$$= 2(1) = 2 \checkmark$$

$$5) (1 + \sin x)^2 + \cos^2 x = 2(1 + \sin x)$$

$$\Rightarrow 1 + 2 \sin x + \sin^2 x + \cos^2 x$$

$$\Rightarrow 1 + 2 \sin x + 1$$

$$\Rightarrow 2 + 2 \sin x$$

$$\Rightarrow 2(1 + \sin x) \checkmark$$

$$6) \sin^4 x - \cos^4 x = 2 \sin^2 x - 1$$

$$= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$$

$$= \sin^2 x - \cos^2 x$$

$$= \sin^2 x - (1 - \sin^2 x)$$

$$= \sin^2 x - 1 + \sin^2 x$$

$$= 2 \sin^2 x - 1 \checkmark$$

PRE - CALCULUS MATH 40S: EXPLAINED!

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## PART V OTHER PROOFS

$$7) (\tan x - 1)^2 = \sec^2 x - 2 \tan x \quad 8) (1 - \sec^2 x)(1 - \sin^2 x) = -\sin^2 x \quad 9) \csc x - \csc^3 x = \frac{-\cos^2 x}{\sin^3 x}$$

$$= (\tan x - 1)(\tan x - 1) \quad (-\tan^2 x)(\cos^2 x) \quad \csc x(1 - \csc^2 x)$$

$$= \tan^2 x - 2 \tan x + 1 \quad -\frac{\sin^2 x \cos^2 x}{\cos^2 x} \quad \csc x(-\cot^2 x)$$

$$= \tan^2 x + 1 - 2 \tan x \quad -\sin^2 x \checkmark \quad \frac{1}{\sin x} \left( -\frac{\cos^2 x}{\sin^2 x} \right)$$

$$\sec^2 x - 2 \tan x \checkmark \quad -\frac{\cos^2 x}{\sin^3 x} \checkmark$$

$$10) \csc^4 x - 1 = \frac{\cos^2 x(1 + \sin^2 x)}{\sin^4 x}$$

$$(\csc^2 - 1)(\csc^2 + 1)$$

$$\cot^2 x (\csc^2 + 1)$$

$$\frac{\cos^2 x}{\sin^2 x} \left( \frac{1}{\sin^2 x} + 1 \right)$$

$$= \frac{\cos^2 x}{\sin^4 x} + \frac{\cos^2 x}{\sin^2 x} \left( \frac{\sin^2 x}{\sin^2 x} \right)$$

$$= \frac{\cos^2 x + \cos^2 x \sin^2 x}{\sin^4 x}$$

$$= \frac{\cos^2 x(1 + \sin^2 x)}{\sin^4 x} \checkmark$$

$$11) \tan^2 x - \cot^2 x = \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}$$

$$\left( \frac{\sin^2 x}{\sin^2 x} \right) \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} \left( \frac{\cos^2 x}{\cos^2 x} \right)$$

$$= \frac{\sin^4 x}{\cos^2 x \sin^2 x} - \frac{\cos^4 x}{\cos^2 x \sin^2 x}$$

$$= \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x}$$

$$= \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x} \checkmark$$

