TRIGONOMETRY II - LESSON TWO

PART IV COMPOUND FRACTIONS & SPECIAL IDENTITIES

Questions: *Prove each of the following:*

1) $\dfrac{\sec x}{\cot x + \tan x} = \sin x$

$= \dfrac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$

$= \dfrac{\frac{1}{\cos x}}{\left(\frac{\cos x}{\cos x}\right)\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\left(\frac{\sin x}{\sin x}\right)}$

$= \dfrac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\cos x \sin x}} = \dfrac{\frac{1}{\cos x}}{\frac{1}{\cos x \sin x}} = \frac{1}{\cos x} \cdot \frac{\cos x \sin x}{1} = \sin x$ ✓

2) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$

$= \dfrac{\sin x + \frac{\sin x}{1} \cdot \frac{\cos x}{\cos x}}{\cos x + 1}$

$= \dfrac{\frac{\sin x \cos x + \sin x}{\cos x}}{\cos x + 1}$

$= \dfrac{\sin x \cos x + \sin x}{\cos x} \cdot \dfrac{1}{\cos x + 1}$

$= \dfrac{\sin x (\cos x + 1)}{\cos x} \cdot \dfrac{1}{\cos x + 1} = \dfrac{\sin x}{\cos x} = \tan x$ ✓

3) $\dfrac{\cos x - \csc x}{\sin x - \sec x} = \cot x$

$= \dfrac{\cos x - \frac{1}{\sin x}}{\frac{\sin x}{1} - \frac{1}{\cos x}}$

$= \dfrac{\frac{\cos x \sin x - 1}{\sin x}}{\frac{\sin x \cos x - 1}{\cos x}}$

$= \dfrac{\cos x \sin x - 1}{\sin x} \cdot \dfrac{\cos x}{\sin x \cos x - 1} = \dfrac{\cos x}{\sin x}$

$= \cot x$ ✓

4) $\dfrac{\sin x + \cos x}{\sec x + \csc x} = \sin x \cos x$

$= \dfrac{\sin x + \cos x}{\left(\frac{\sin x}{\sin x}\right)\frac{1}{\cos x} + \frac{1}{\sin x}\left(\frac{\cos x}{\cos x}\right)}$

$= \dfrac{\sin x + \cos x}{\frac{\sin x + \cos x}{\sin x \cos x}}$

$= \dfrac{\sin x + \cos x}{1} \cdot \dfrac{\sin x \cos x}{\sin x + \cos x}$

$= \sin x \cos x$ ✓

5) $\dfrac{\tan x - \sin x}{\tan x \sin x} = \dfrac{1 - \cos x}{\sin x}$

$= \dfrac{\frac{\sin x}{\cos x} - \frac{\sin x}{1}\left(\frac{\cos x}{\cos x}\right)}{\frac{\sin x}{\cos x} \cdot \frac{\sin x}{1}}$

$= \dfrac{\frac{\sin x - \sin x \cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}}$

$= \dfrac{\sin x - \sin x \cos x}{\cos x} \cdot \dfrac{\cos x}{\sin^2 x}$

$= \dfrac{\sin x (1 - \cos x)}{\cos x} \cdot \dfrac{\cos x}{\sin^2 x}$

$= \dfrac{1 - \cos x}{\sin x}$ ✓

6) $\dfrac{1 + \cos x}{\tan x + \sin x} = \cot x$

$= \dfrac{1 + \cos x}{\frac{\sin x}{\cos x} + \frac{\sin x}{1}\left(\frac{\cos x}{\cos x}\right)}$

$= \dfrac{1 + \cos x}{\frac{\sin x + \sin x \cos x}{\cos x}}$

$= \dfrac{1 + \cos x}{1} \cdot \dfrac{\cos x}{\sin x (1 + \cos x)}$

$= \dfrac{\cos x}{\sin x} = \cot x$ ✓

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7) $\frac{1+\tan^2 x}{1+\cot^2 x} = \tan^2 x$

$= \frac{\sec^2 x}{\csc^2 x}$

$= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}$

$= \frac{1}{\cos^2 x} \cdot \sin^2 x$

$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$ ✓

8) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

$= \cos^2 x + \sin^2 x$

$= 1$ ✓

9) $\frac{1+\tan x}{1+\cot x} = \tan x$

$\frac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}}$

$= \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}}$

$= \frac{\cos x + \sin x}{\cos x} \cdot \frac{\sin x}{\sin x + \cos x} = \frac{\sin x}{\cos x} = \tan x$ ✓

10) $\frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1} = 2\cot^2 x$

$= \frac{\cos x}{\sec x - 1}\frac{(\sec x + 1)}{(\sec x + 1)} + \frac{\cos x}{\sec x + 1}\frac{(\sec x - 1)}{(\sec x - 1)}$

$= \frac{\cos x \sec x + \cos x + \cos x \sec x - \cos x}{(\sec x - 1)(\sec x + 1)}$

$= \frac{2\cos x \sec x}{(\sec x - 1)(\sec x + 1)} = \frac{2\cos x \sec x}{\sec^2 x - 1}$

$= \frac{2\cos x \sec x}{\tan^2 x} = \frac{2\cos x \frac{1}{\cos x}}{\tan^2 x} = \frac{2}{\tan^2 x} = 2\cot^2 x$

11) $\frac{\tan x}{1+\tan x} = \frac{\sin x}{\sin x + \cos x}$

$\frac{\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} \Rightarrow \frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}$

$\Rightarrow \frac{\frac{\sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}}$

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\cos x + \sin x}$

$\Rightarrow \frac{\sin x}{\cos x + \sin x}$ ✓

12) $\frac{\sin^2 x}{1-\sin x} + \frac{\sin^2 x}{1+\sin x} = 2\tan^2 x$

$= \frac{\sin^2 x}{1-\sin x}\frac{(1+\sin x)}{(1+\sin x)} + \frac{\sin^2 x}{1+\sin x}\frac{(1-\sin x)}{(1-\sin x)}$

$= \frac{\sin^2 x + \sin^3 x}{(1-\sin x)(1+\sin x)} + \frac{\sin^2 x - \sin^3 x}{(1+\sin x)(1-\sin x)}$

$= \frac{\sin^2 x + \sin^3 x + \sin^2 x - \sin^3 x}{(1-\sin x)(1+\sin x)}$

$= \frac{2\sin^2 x}{(1-\sin x)(1+\sin x)}$

$= \frac{2\sin^2 x}{1-\sin^2 x} \Rightarrow \frac{2\sin^2 x}{\cos^2 x} = 2\tan^2 x$ ✓