

TRIGONOMETRY II - LESSON TWO

PART III THREE SPECIAL IDENTITIES

Questions: Use the special identities to do each of the following proofs.

1) $\sec x - \tan x \sin x = \cos x$

$$\begin{aligned} &= \frac{1}{\cos x} - \left(\frac{\sin x}{\cos x}\right) \sin x \\ &= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} \\ &= \frac{1 - \sin^2 x}{\cos x} \\ &= \frac{\cos^2 x}{\cos x} \\ &= \cos x \checkmark \end{aligned}$$

3) $\tan x + \cot x = \sec x \csc x$

$$\begin{aligned} &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\ &= \frac{\sin x}{\cos x} \left(\frac{\sin x}{\sin x}\right) + \frac{\cos x}{\sin x} \left(\frac{\cos x}{\cos x}\right) \\ &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \\ &= \frac{1}{\sin x \cos x} \\ &= \sec x \csc x \checkmark \end{aligned}$$

5) $\sec x - \cos x = \tan x \sin x$

$$\begin{aligned} &= \frac{1}{\cos x} - \frac{\cos x}{1} \\ &= \frac{1}{\cos x} - \frac{\cos x}{1} \left(\frac{\cos x}{\cos x}\right) \\ &= \frac{1 - \cos^2 x}{\cos x} \\ &= \frac{\sin^2 x}{\cos x} \\ &= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1} \\ &= \tan x \sin x \checkmark \end{aligned}$$

2) $\cos x + \tan x \sin x = \sec x$

$$\begin{aligned} &= \cos x + \left(\frac{\sin x}{\cos x}\right) \sin x \\ &= \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} \\ &= \left(\frac{\cos x}{\cos x}\right) \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos x} \\ &= \frac{1}{\cos x} = \sec x \checkmark \end{aligned}$$

4) $1 + \tan^2 x = \sec^2 x$

$$\begin{aligned} &= 1 + \frac{\sin^2 x}{\cos^2 x} \\ &= \left(\frac{\cos^2 x}{\cos^2 x}\right) + \frac{\sin^2 x}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \checkmark \end{aligned}$$

6) $\sin x + \cot x \cos x = \csc x$

$$\begin{aligned} &= \sin x + \left(\frac{\cos x}{\sin x}\right) \cos x \\ &= \frac{\sin x}{1} + \frac{\cos^2 x}{\sin x} \\ &= \frac{\sin x}{1} \left(\frac{\sin x}{\sin x}\right) + \frac{\cos^2 x}{\sin x} \\ &= \frac{\sin^2 x + \cos^2 x}{\sin x} \\ &= \frac{1}{\sin x} = \csc x \checkmark \end{aligned}$$

TRIGONOMETRY II - LESSON TWO

PART III

THREE SPECIAL IDENTITIES

7) $\sec^2 x - 1 = \sin^2 x \sec^2 x$

$$= \frac{1}{\cos^2 x} - \frac{1}{1} \left(\frac{\cos^2 x}{\cos^2 x} \right)$$

$$= \frac{1 - \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \sin^2 x \left(\frac{1}{\cos^2 x} \right)$$

$$= \sin^2 x \sec^2 x \checkmark$$

8) $1 - \csc^2 x = -\cot^2 x$

$$= 1 - \frac{1}{\sin^2 x}$$

$$= \frac{1}{1} \left(\frac{\sin^2 x}{\sin^2 x} \right) - \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x - 1}{\sin^2 x}$$

$$= \frac{-\cos^2 x}{\sin^2 x}$$

$$= -\cot^2 x \checkmark$$

9) $\csc x - \sin x = \cos x \cot x$

$$= \frac{1}{\sin x} - \frac{\sin x}{1} \left(\frac{\sin x}{\sin x} \right)$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \frac{\cos x}{\sin x} (\cos x)$$

$$= \cot x \cos x \checkmark$$

10) $1 - \sec^2 x = -\tan^2 x$

$$= 1 - \frac{1}{\cos^2 x}$$

$$= \left(\frac{\cos^2 x}{\cos^2 x} \right) \frac{1}{1} - \frac{1}{\cos^2 x}$$

$$= \frac{\cos^2 x - 1}{\cos^2 x}$$

$$= \frac{-\sin^2 x}{\cos^2 x}$$

$$= -\tan^2 x \checkmark$$

