

TRIGONOMETRY II - LESSON TWO

PART 1

MULTIPLICATION & DIVISION IDENTITIES

For each of the following, write an algebraic proof.

1) Prove: $\cot x \tan x = 1$

$$\begin{aligned} &= \left(\frac{\cos x}{\sin x}\right) \left(\frac{\sin x}{\cos x}\right) \\ &= 1 \quad \checkmark \end{aligned}$$

2) Prove: $\csc x \cos x = \cot x$

$$\begin{aligned} &= \left(\frac{1}{\sin x}\right) \cos x \\ &= \frac{\cos x}{\sin x} \\ &= \cot x \quad \checkmark \end{aligned}$$

3) Prove: $\frac{\sin x}{\tan x} = \cos x$

$$\begin{aligned} &= \frac{\sin x}{\frac{\sin x}{\cos x}} \\ &= \sin x \cdot \left(\frac{\cos x}{\sin x}\right) = \cos x \quad \checkmark \end{aligned}$$

4) Prove: $\frac{1}{\cot x \cos x \tan x} = \sec x$

$$\begin{aligned} &= \frac{1}{\left(\frac{\cos x}{\sin x}\right) (\cos x) \left(\frac{\sin x}{\cos x}\right)} \\ &= \frac{1}{\cos x} \\ &= \sec x \quad \checkmark \end{aligned}$$

Identities will always have the following two properties:

1) If you graph the left and right sides, you will obtain exactly the same graph.

2) If you plug in the same angle for x on both sides, you will obtain exactly the same number.

TRIGONOMETRY II - LESSON TWO

PART I

MULTIPLICATION & DIVISION IDENTITIES

5) Prove: $\frac{\tan x}{\csc x} = \frac{\sin^2 x}{\cos x}$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\frac{1}{\sin x}}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1}$$

$$= \frac{\sin^2 x}{\cos x} \quad \checkmark$$

6) Prove: $\frac{\tan x}{\sec x} = \sin x$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\frac{1}{\cos x}}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}$$

$$= \sin x \quad \checkmark$$

7) Prove: $\frac{\cos^2 x}{\cot x} = \sin x \cos x$

$$= \frac{\cos^2 x}{\frac{\cos x}{\sin x}}$$

$$= \cos^2 x \cdot \frac{\sin x}{\cos x}$$

$$= \sin x \cos x \quad \checkmark$$

8) Prove: $\frac{\sec x \csc x}{\cot x} = \sec^2 x$

$$= \frac{\frac{1}{\cos x} \cdot \frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$$

$$= \frac{1}{\cos x \sin x} \cdot \frac{\sin x}{\cos x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x \quad \checkmark$$

9) Prove: $\frac{\sec x \csc x}{\csc^2 x} = \tan x$

$$= \frac{\sec x}{\csc x}$$

$$= \frac{1}{\cos x} \cdot \frac{1}{\frac{1}{\sin x}}$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{1}$$

$$= \frac{\sin x}{\cos x} = \tan x \quad \checkmark$$

10) Prove: $\frac{\tan^2 x \cos x}{2 \sec x} = \frac{1}{2} \sin^2 x$

$$= \frac{\frac{\sin^2 x}{\cos^2 x} \cdot \cos x}{\frac{2}{\cos x}}$$

$$= \frac{\sin^2 x}{\cos x} \cdot \frac{\cos x}{2}$$

$$= \frac{\sin^2 x}{2} = \frac{1}{2} \sin^2 x \quad \checkmark$$