The Fundamental theorem of Algebra

Sec 2.5 pg 187 # 9-14, 37, 39, 41, 43, 49, 53

#9) $f(x) = (x^3 - 4x^2) + (x - 4)$

$= x^2(x-4) + 1(x-4)$

$= (x^2+1)(x-4)$

Zeros: $4, i, -i$

The only real zero of $f(x)$ is $x=4$. This corresponds w/the x-int. of $(4,0)$ on the graph.

#10) $f(x) = (x^3 - 4x^2) - (4x + 16)$

$= x^2(x-4) - 4(x-4)$

$= (x-4)(x^2-4)$

$= (x-4)(x+2)(x-2)$

Zeros: $4, -2, 2$

There are 3 real zeros $4, -2, 2$ that correspond w/ x-int on the graph.

#11) $f(x) = x^4 + 4x^2 + 4$

$= (x^2+2)(x^2+2) \longrightarrow \sqrt{x^2} = \sqrt{-2}$

$= (x^2+2)^2 \qquad x = \pm i\sqrt{2}$

Zeros: $\pm i\sqrt{2}, \pm i\sqrt{2}$

$f(x)$ has no real zeros and the graph of $f(x)$ has no x-intercepts.

#12) $f(x) = x^4 - 3x^2 - 4$

$x^2 = 4 \qquad = (x^2-4)(x^2+1) \qquad x^2 = -1$

$x = \pm 2 \qquad\qquad x = \pm i$

Zeros: $2, -2, i, -i$

There are 2 real zeros 2 and -2, that correspond w/x-int. on the graph.

#13) $h(x) = x^2 - 4x + 1$

$x = \dfrac{+4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{16-4}}{2} \qquad \sqrt{12}$

$= \dfrac{4 \pm \sqrt{12}}{2} \longrightarrow \sqrt{4}\sqrt{3}$

$= \dfrac{4 \pm 2\sqrt{3}}{2}$

$x = 2 \pm \sqrt{3}$

$h(x) = [x - (2+\sqrt{3})][x - (2-\sqrt{3})]$

$\boxed{h(x) = (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})}$

#14) $g(x) = x^2 + 10x + 23$

$x = \dfrac{-10 \pm \sqrt{(10)^2 - 4(1)(23)}}{2(1)}$

$= \dfrac{-10 \pm \sqrt{100-92}}{2} \qquad \sqrt{8}$

$= \dfrac{-10 \pm \sqrt{8}}{2} \longrightarrow \sqrt{4}\sqrt{2}$

$= \dfrac{-10 \pm 2\sqrt{2}}{2}$

$x = -5 \pm \sqrt{2}$

$g(x) = [x - (-5+\sqrt{2})][x - (-5-\sqrt{2})]$

$\boxed{g(x) = (x + 5 - \sqrt{2})(x + 5 + \sqrt{2})}$

Cont. Sec 2.5 # 37, 39, 41, 43, 49, 53.

#37) $f(x) = x^3 - 11x + 150$

150
1·150
2·75
3·50
5·30
6·25
10·15

a) $= (x+6)(x^2 - 6x + 25)$

$$x = \frac{+6 \pm \sqrt{(6)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 100}}{2}$$

$$= \frac{6 \pm \sqrt{-64}}{2}$$

$$= \frac{6 \pm 8i}{2}$$

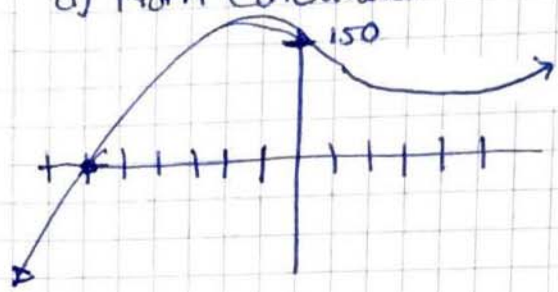
$$x = 3 \pm 4i$$

zeros = $-6, 3+4i, 3-4i$

b) $f(x) = (x+6)(x-3+4i)(x-3-4i)$

c) x-intercepts : $(-6, 0)$

d) From Calculater



#39) $f(x) = x^4 + 25x^2 + 144$

144
1·144
2·72
3·48
4·36
6·24
8·18
9·16
12·12

a) $= (x^2 + 9)(x^2 + 16)$

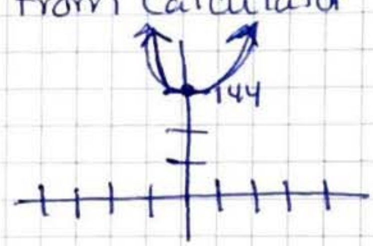
$x^2 = -9$, $x = \pm 3i$ $\quad$ $x^2 = -16$, $x = \pm 4i$

Zeros: $3i, -3i, 4i, -4i$

b) $f(x) = (x+3i)(x-3i)(x+4i)(x-4i)$

c) No x-intercepts

d) From Calculator



#41) $1, 5i, -5i$

$i^2 = -1$

$f(x) = (x-1)(x-5i)(x+5i)$

$= (x-1)(x^2 + \cancel{5ix} - \cancel{5ix} - 25i^2)$

$= (x-1)(x^2 + 25)$

$= x^3 + 25x - x^2 - 25$

$f(x) = x^3 - x^2 + 25x - 25$

$f(x) = a(x^3 - x^2 + 25 - 25)$

where "a" is non-zero #

#43) $2, 4+i, 4-i$

$f(x) = (x-2)(x-4-i)(x-4+i)$

$= (x-2)(x^2 - 4x + \cancel{ix} - 4x + 16 - \cancel{4i} - \cancel{ix} + \cancel{4i} - \overset{+1}{i^2})$

$= (x-2)(x^2 - 8x + 17)$

$= x^3 - 8x^2 + 17x - 2x^2 + 16x - 34$

$f(x) = x^3 - 10x + 33x - 34$

$f(x) = a(x^3 - 10x + 33x - 34)$

Where "a" is a non-zero #

#49) $f(x)=x^4-6x^2-7$

a) Rationals

$f(x)=(x^2-7)(x^2+1)$

b) Reals

$f(x)=(x-\sqrt{7})(x+\sqrt{7})(x^2+1)$

c) Complete factore form

$f(x)=(x-\sqrt{7})(x+7)(x+i)(x-i)$

#53) $f(x)=2x^3+3x^2+50x+75$; Zero $5i$

So, since $5i$ is zero then $-5i$ is also a zero.

$$\begin{array}{r|rrrr} 5i & 2 & 3 & 50 & 75 \\ & \downarrow & 10i & -50+15i & -75 \\ \hline & 2 & 3+10i & 15i & 0 \end{array}$$

$$\begin{array}{r|rrr} -5i & 2 & 3+10i & 15i \\ & \downarrow & -10i & -15i \\ \hline & 2 & 3 & 0 \end{array}$$

$f(x)=(x-5i)(x+5i)(2x+3)$

$\boxed{\text{Zeros: } 5i, -5i, -3/2}$