Real Zeros of Polynomial Functions

Sec 2.3 pg #170 #9, 19, 29, 35, 39, 47, 51-55 odd, 65, 93

#9) $4x^3-7x^2-11x+5$ by $4x+5$

$$\begin{array}{r|l} & x^2-3x+1 \\ \hline 4x+5 & 4x^3-7x^2-11x+5 \\ & -4x^3-5x^2 \\ \hline & -12x^2-11x \\ & +12x^2+15x \\ \hline & 4x+5 \\ & -4x-5 \\ \hline & 0 \end{array}$$

#19) $(3x^3-10x^2+12x-22) \div (x-4)$

$$\begin{array}{r|rrrr} 4 & 3 & -10 & 12 & -22 \\ & \downarrow & 12 & 8 & 80 \\ \hline & 3 & 2 & 20 & 58 \end{array}$$

$$\boxed{3x^2+2x+20+\frac{58}{x-4}}$$

#29) $f(x) = x^3-x^2-14x+11$, $k=4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & \downarrow & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x-4)(x^2-3x-2)+3$

$f(4) = (4-4)(4^2-3(4)-2)+3$

$= (0)(26)+3$

$f(4) = 3$, $(4,3)$

#35) $f(x) = 4x^3-13x+10$

a) $f(1)$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -13 & 10 \\ & \downarrow & 4 & 4 & -9 \\ \hline & 4 & 4 & -9 & 1 \end{array}$$

$\boxed{f(1)=1}$

b) $f(-2)$

$$\begin{array}{r|rrrr} -2 & 4 & 0 & -13 & 10 \\ & \downarrow & -8 & 16 & -6 \\ \hline & 4 & -8 & 3 & 4 \end{array}$$

$\boxed{f(-2)=4}$

c) $f(\frac{1}{2})$

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & 0 & -13 & 10 \\ & \downarrow & 2 & 1 & -6 \\ \hline & 4 & 2 & -12 & 4 \end{array}$$

$\boxed{f(\frac{1}{2})=4}$

d) $f(8)$

$$\begin{array}{r|rrrr} 8 & 4 & 0 & -13 & 10 \\ & \downarrow & 32 & 256 & 1944 \\ \hline & 4 & 32 & 243 & 1954 \end{array}$$

$\boxed{f(8)=1954}$

#39) $x^3-7x+6=0$; $x=2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & \downarrow & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^3-7x+6 = (x-2)(x^2-2x-3)$

$=(x-2)(x+3)(x-1)$

Zeros: $x = 2, -3, 1$

Cont. #47, 51 – 55 odd, 65, 93

#47) $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$; Factors $(x-5), (x+4)$

a)

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & \downarrow & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \checkmark \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & \downarrow & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \checkmark \end{array}$$

b) find remaining factors of f.

$x^2 - 3x + 2$

$(x-2)(x-1)$

Reamaning factors: $(x-2), (x-1)$

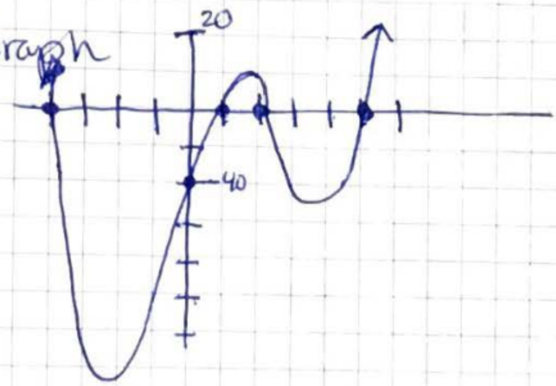
c) write complete factorization of f.

$f(x) = (x-5)(x+4)(x-2)(x-1)$

d) Real zeros:

$x = 5, -4, 2, 1$

e) Graph



#53) $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

$\frac{P}{q} = \frac{\pm 1, \pm 3, \pm 5, \pm 9, \pm 15}{\pm 1, \pm 2}$

Possible zeros $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2}$

$$\begin{array}{r|rrrrr} 3 & 2 & -17 & 35 & 9 & -45 \\ & \downarrow & 6 & -33 & 6 & 45 \\ \hline & 2 & -11 & 2 & 15 & 0 \end{array}$$

$f(x) = (x-3)(2x^3 - 11x^2 + 2x + 15)$

$$\begin{array}{r|rrrr} 5 & 2 & -11 & 2 & 15 \\ & \downarrow & 10 & -5 & -15 \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

$f(x) = (x-3)(x-5)(2x^2 - x - 3)$ ← Factor

$f(x)\ (x-3)(x-5)(2x-3)(x+1)$

Zeros on graph $x = 3, 5, \frac{3}{2}, -1$

#51) $f(x) = x^3 + 3x^2 - x - 3$

Possible Rational Zeros $\frac{\text{Factors (P)}}{\text{Factors (q)}} = \frac{\pm 1, \pm 3}{\pm 1}$

→ $\pm 1, \pm 3$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -1 & -3 \\ & \downarrow & -3 & 0 & 3 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$(x+3)(x^2-1)$

$(x+3)(x-1)(x+1)$

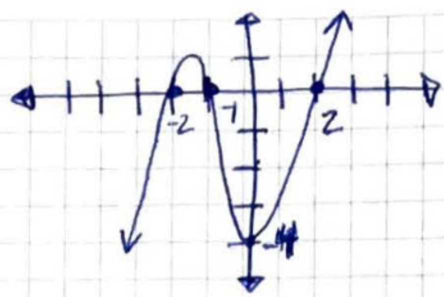
NOTE: I did synthetic but I think it was asking to look at Calc. to zero if zeros matched.

Zeros on graph: $x = -3, -1, 1$

#55) $f(x) = x^3 + x^2 - 4x - 4$

a) $\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1}$

Possible zeros: $\pm 1, \pm 2, \pm 4$

b) 

c) Zeros from graph(b)

$x = -2, -1, 2$

#65) $x^4 - 13x^2 - 12x = 0$

$x(x^3 - 13x - 12) = 0$

$\underline{\underline{x=0}}$ is a soln

Possible zeros $\pm 1, \pm 2 \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -13 & -12 \\ & \downarrow & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & -24 \end{array} \quad \text{not zero}$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & \downarrow & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$x(x-1)(x^2 - x - 12) = 0$

$x(x-1)(x-4)(x+3) = 0$

$\boxed{\text{Zeros}: x = 0, 1, 4, -3}$

#93)
If $(7x+4)$ is a factor of some polynomial function f, then $4/7$ is a root of f.

False, $-4/7$ is a zero of f.