

$$x^2 + 2x + 3 = 0$$

$$x^2 + 2x + 1 = -3 + 1$$

$$(x+1)^2 = -2$$

$$(x+1)^2 + 2 = 0 \quad \sqrt{(-1, 2)}$$

Review Section 1.1 part 3 and 4

Name: Key
Pre-Calculus

Part I. Carefully graph each of the following, then evaluate

1. $h(x) = \begin{cases} x+5, & x < -2 \\ x^2+2x+3, & x \geq -2 \end{cases}$

$h(3) = 18$
 $h(-4) = 1$
 $h(-2) = 3$

3. $g(x) = \begin{cases} x^2-1, & x \leq 0 \\ 2x-1, & 0 < x \leq 2 \\ 3, & x > 2 \end{cases}$

$g(-2) = 3$
 $g(0) = -1$
 $g(5) = 3$

2. $m(x) = \begin{cases} 5, & x \leq -3 \\ -2x-3, & x > -3 \end{cases}$

$m(-4) = 5$
 $m(0) = -3$
 $m(3) = -9$

4. $p(x) = \begin{cases} 2, & x \leq -3 \\ 2x-1, & -3 < x < 2 \\ -3, & x \geq 2 \end{cases}$

$p(0) = -1$
 $p(-3) = -7$
 $p(2) = -3$

Part II. Write equations for the piecewise functions whose graphs are shown below. Assume that the units are 1 for every tick mark.

5.

$$f(x) = \begin{cases} x+5, & x < -2 \\ -2, & x \geq -2 \end{cases}$$

6.

$$f(x) = \begin{cases} x+2, & x \leq -1 \\ x+3, & -1 < x < 1 \\ x+1, & x \geq 1 \end{cases}$$

Part III: Write the set and interval notation for the domain of each function.

7. $f(x) = x+5$ $D: \{x \in \mathbb{R}\}$, $D: (-\infty, \infty)$
8. $f(x) = \frac{1}{2x+5}$ $2x+5 \neq 0$
 $2x \neq -5$
 $x \neq -5/2$ $D: \{x \in \mathbb{R} \mid x \neq -5/2\}$, $D: (-\infty, -5/2) \cup (-5/2, \infty)$
9. $f(x) = \sqrt{x+4}$
 $x+4 \geq 0$
 $x \geq -4$ $D: \{x \in \mathbb{R} \mid x \geq -4\}$, $D: [-4, \infty)$
10. $f(x) = \frac{1}{\sqrt{x+3}}$
 $x+3 > 0$
 $x > -3$ $D: \{x \in \mathbb{R} \mid x > -3\}$, $D: (-3, \infty)$
11. $f(x) = \frac{\sqrt{x-2}}{x}$
 $x-2 \geq 0$
 $x \geq 2$ $D: \{x \in \mathbb{R} \mid x \geq 2\}$, $D: [2, \infty)$
12. Write the interval notation of this domain in set notation.
a. $(-\infty, 8) \cup (12, \infty)$ $b. (-3, 1]$
Set Notations: $\{x \in \mathbb{R} \mid x < 8, x > 12\}$ $\{x \in \mathbb{R} \mid -3 < x \leq 1\}$

For problem 13-14, please give the name of the parent function and describe the transformation represented. Then sketch the graph of the given function.

13. $g(x) = (x+1)^2 - 1$
Parent Function: Quadratic
Transformation: 1 unit left and 1 unit up

14. $f(x) = \frac{1}{2}|x-1|$
Parent Function: Absolute Value
Transformation: vertically compressed by 1/2 & 1 unit right

Give the parent function and write the equation of the transformation function $f(x)$.

15. Absolute value—vertical shift up 5, horizontal shift right 3

$$y = |x| ; f(x) = |x-3| + 5$$

16. Quadratic—vertical stretch by 5, horizontal shift left 8

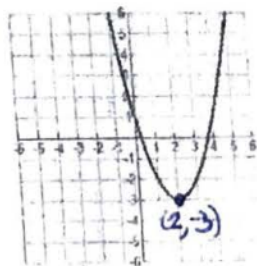
$$y = x^2 ; f(x) = 5(x+8)^2$$

17. Square root—vertical shift down 2

$$y = \sqrt{x} ; f(x) = \sqrt{x} - 2$$

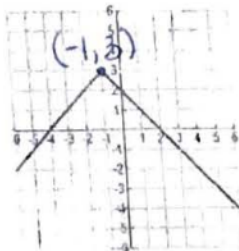
In problems 18-19, the transformation of a parent function is shown. All graphs have been only vertically translated, horizontally translated, and/or reflected over the x-axis. Write the equation of each function.

18.



$$y = (x-2)^2 - 3$$

19.



$$y = -|x+1| + 3$$

20. Given $f(x) = 2x^2 - x$, find the following and simplify.

(a) $f(x+h)$

$$\begin{aligned} &= 2(x+h)^2 - (x+h) \\ &= 2(x^2 + 2xh + h^2) - (x+h) \\ &= 2x^2 + 4xh + 2h^2 - x - h \\ &= 2x^2 - x + 4xh - h + 2h^2 \end{aligned}$$

(b) $f(x+h) - f(x)$

$$\begin{aligned} &2x^2 - x + 4xh - h + 2h^2 - (2x^2 - x) \\ &= 4xh - h + 2h^2 \end{aligned}$$

(c) $\frac{f(x+h) - f(x)}{h}$

$$\begin{aligned} &\frac{4xh - h + 2h^2}{h} \\ &= 4x - 1 + 2h \end{aligned}$$

21. Given $C(x) = 2x^2 - 4x + 3$, find and simplify $\frac{C(x+h) - C(x)}{h}$.

$$\begin{aligned} C(x+h) &= 2(x+h)^2 - 4(x+h) + 3 \\ &= 2(x^2 + 2xh + h^2) - 4x - 4h + 3 \\ &= 2x^2 + 4xh + 2h^2 - 4x - 4h + 3 \end{aligned}$$

$$\begin{aligned} \frac{C(x+h) - C(x)}{h} &= \frac{2x^2 + 4xh + 2h^2 - 4x - 4h + 3 - (2x^2 - 4x + 3)}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 - 4x - 4h + 3 - 2x^2 + 4x - 3}{h} \\ &= \frac{4xh + 2h^2 - 4h}{h} \rightarrow \frac{h(4x + 2h - 4)}{h} \end{aligned}$$

Perform the indicated operation.

1) $f(x) = x - 5x$
 $g(x) = 2x - 1$
 Find $f(g^{-1}(x))$

3) $h(n) = 3n - 4$
 $g(n) = n - 3$
 Find $(h \circ g)^{-1}(n)$

5) $g(x) = -4x - 4$
 $h(x) = x - 2$
 Find $(g^{-1} \circ h)(-7)$

2) $g(n) = -n^2 - 2n + 1$
 $f(n) = n - 1$
 Find $g(f(n))$

4) $g(a) = 4a - 3$
 $h(a) = 2a + 4$
 Find $g^{-1}(h^{-1}(a))$

6) $f(t) = 4t - 5$
 Find $f(f(4))$

State if the given functions are inverses.

7) $g(x) = -2x - 2$
 $f(x) = -2x + 2$

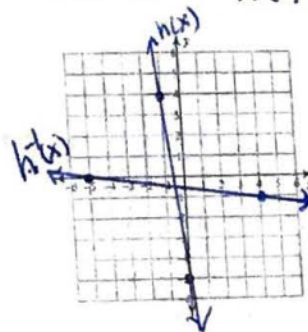
9) $f(x) = -\frac{1}{4}x + \frac{1}{4}$
 $g(x) = -5x + 3$

8) $f(x) = \frac{1}{2}x$
 $g(x) = 2x$

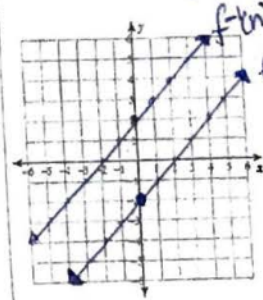
10) $f(n) = 4n - 4$
 $g(n) = \frac{-n-3}{6}$

Find the inverse of each function. Then graph the function and its inverse.

11) $h(x) = -9x - 5$ $h^{-1}(x) = \frac{x+5}{-9}$
 $= -\frac{x+5}{9}$



12) $f(n) = n - 2$ $f^{-1}(n) = n + 2$



look at 3rd page

1) Find $f(g^{-1}(x))$

$$f(x) = x - 5x \Rightarrow 4x$$

$$g^{-1}(x) = \frac{x+1}{2}$$

$$f(g^{-1}(x)) = -4 \left(\frac{x+1}{2} \right)$$

$$= -2(x+1)$$

$$= \boxed{-2x-2}$$

2) Find $g(f(n)) = -(n-1)^2 - 2(n-1) +$

$$= -(n^2 - 2n + 1) - 2n + 2$$

$$g(n) = -n^2 - 2n + 1 = -n^2 + 2n - 1 - 2n + 2 +$$

$$f(n) = n - 1$$

$$= \boxed{-n^2 + 2}$$

3) Find $(h \circ g)^{-1}(n) = h^{-1}(g^{-1}(n))$

$$h(n) = 3n - 4$$

$$h^{-1}(n) = \frac{n+4}{3}$$

$$= \frac{(n+3)+4}{3}$$

$$g(x) = n - 3$$

$$g^{-1}(x) = n + 3$$

$$= \boxed{\frac{n+7}{3}}$$

4) Find $g^{-1}(h^{-1}(a))$

$$g(a) = 4a - 3$$

$$h(a) = 2a + 4$$

$$g^{-1}(a) = \frac{a+3}{4}$$

$$h^{-1}(a) = \frac{a-4}{2}$$

$$= \frac{\left(\frac{a-4}{2}\right) + 3 \cdot 2}{4}$$

$$= \frac{a-4+6}{2 \cdot 2} \Rightarrow \frac{a+2}{4} \cdot \frac{1}{4}$$

$$= \boxed{\frac{a+2}{8}}$$

5) Find $(g^{-1} \circ h)(-7) \Rightarrow g^{-1}(h(-7))$

$$g(x) = -4x - 4$$

$$g^{-1}(x) = \frac{x+4}{-4}$$

$$h(x) = x - 2$$

$$h(-7) = -7 - 2 = \underline{-9}$$

$$g^{-1}(h(-7)) = \frac{-9+4}{-4}$$

$$= \frac{-5}{-4} = \boxed{\frac{5}{4}}$$

#8) $f(g(x)) = f(2x)$ $g(f(x)) = g(\frac{1}{2}x)$
 $= \frac{1}{2}(2x)$ $= 2(\frac{1}{2}x)$
 $= x \checkmark$ $= x \checkmark$
 yes, inverse

#9) $f(x) = -\frac{1}{4}x + \frac{1}{4}$ $f(g(x)) = f(-5x+3)$
 $g(x) = -5x+3$ $= -\frac{1}{4}(-5x+3) + \frac{1}{4}$
 $= \frac{5x}{4} + \frac{3}{4} + \frac{1}{4}$
 $= \frac{5x}{4} + \frac{4}{4}$
 $= \frac{5x}{4} + 1$ NO
 NOT Inverse
 b/c NOT (\Rightarrow) to x

#10) $f(x) = 4n - 4$ $f(g(x)) = f(n-3)$
 $g(x) = \frac{-n-3}{6}$ $= \frac{4(-n-3)-4}{6}$
 NOT an Inverse $= \frac{-4n-12-4}{6}$
 b/c NOT (\Rightarrow) to x $= \frac{-4n-12-4}{6}$
 $= \frac{-4n-12-4}{6}$
 $= \frac{-4n-12-24}{6}$

$$\frac{-4n - 36}{6}$$

or

$$= -\frac{2n}{3} - 6$$