Section 1.1 Part 2

Functions: Difference Quotient

Objectives: Given a function students will be able to find the difference quotient.

Study Problems Page 84 #79-85 odd

b. $f(x+2)$

$\underbrace{}_{x}$

c. $\dfrac{f(x+2)-f(x)}{2}$

b) $f(x+2) = (x+2)^2 + (x+2) - 1$

$= x^2 + 4x + 4 + x + 2 - 1$

$= x^2 + 5x + 5$

c) $\dfrac{f(x+2) - f(x)}{2}$

$= \dfrac{x^2 + 5x + 5 - (x^2 + x - 1)}{2}$

$= \dfrac{x^2 + 5x + 5 - x^2 - x + 1}{2}$

$= \dfrac{4x + 6}{2}$

$= \boxed{2x + 3}$

Example 2

Evaluate the functions for the given values.

$h(x) = 7 - 5x$

a. $h(1)$

b. $h(x+1)$

c. $\dfrac{h(x) - h(1)}{x - 1}$

a) $h(1) = 7 - 5(1)$
$= \boxed{2}$

b) $h(x+1) = 7 - 5(x+1)$
$= 7 - 5x - 5$
$= \boxed{-5x + 2}$

c) $\rightarrow = \dfrac{7 - 5x - 2}{x - 1}$

$= \dfrac{-5x + 5}{x - 1} = \dfrac{-5(x - 1)}{x - 1}$

$= \boxed{-5}$

Example 4

Evaluate the difference quotient for the function.

$$f(x) = x^2 + 9x + 6$$

Difference Quotient

$$\frac{f(5+h) - f(5)}{h} \quad \text{where } h \neq 0$$

$$f(5+h) = (5+h)^2 + 9(5+h) + 6$$
$$= 25 + 10h + h^2 + 45 + 9h + 6$$
$$= h^2 + 19h + 76$$

$$f(5) = (5)^2 + 9(5) + 6$$
$$= 25 + 45 + 6$$
$$= 76$$

$$\frac{f(5+h) - f(5)}{h} = \frac{h^2 + 19h + 76 - 76}{h}$$

$$= \frac{h^2 + 19h}{h}$$

$$= \frac{h(h + 19)}{h} = \boxed{h + 19}$$

Example 5

Evaluate the difference quotient for the function.

$$f(x) = 2x - x^2 \ \text{ for } \ \frac{f(4+h) - f(4)}{h}, h \neq 0$$

$f(4+h) = 2(4+h) - (4+h)^2$
$= 8 + 2h - (16 + 8h + h^2)$
$= 8 + 2h - 16 - 8h - h^2$
$= -h^2 - 6h - 8$

$f(4) = 2(4) - (4)^2$
$= 8 - 16$
$= -8$

$$\underline{f(4+h) - f(4)} = \frac{-h^2 - 6h - 8 - (-8)}{h}$$

$$= \frac{-h^2 - 6h}{h}$$

$$= \frac{h(-h - 6)}{h} = \boxed{-h - 6}$$