Section 5.1

Part 4

Using Trigonometric Identities

Objective: Given an **equation** students will be able to prove/verify the equation is true by using the fundamental trig identities.

Study Problems

Trigonometry II Part 4 wks

Type IV: compound fractions

Next we'll look at compound fractions. Everything here is basically the same as in the section, just be sure to follow your rules for dividing fractions & watch for special ider

$$\frac{\frac{1}{3} + \frac{x}{1}\cdot\frac{3}{3}}{\frac{x}{x}\cdot\frac{1}{2} + \frac{1}{x}\cdot\frac{2}{2}}$$

$$\frac{\frac{1+3x}{3}}{\frac{x+2}{2x}}$$

$$\frac{1+3x}{3} \cdot \frac{2x}{x+2}$$

$$\frac{2x(1+3x)}{3(x+2)} = \frac{2x+6x^2}{3x+6}$$

Example

Use the trig identities to transform one side of the equation into the other.

$$\frac{\tan^2 x + 1}{\csc^2 x - 1} = \sec^2 x \tan^2 x$$

$$\frac{\sec^2 x}{\cot^2 x}$$

$$\sec^2 x \left(\frac{1}{\cot^2 x}\right)$$

$$\sec^2 x \tan^2 x$$

other.

$$\frac{\tan^2 x+1}{\csc^2 x-1}$$

$$=\frac{\dfrac{\sin^2 x}{\cos^2 x}+1}{\dfrac{1}{\sin^2 x}-1}$$

$$=\frac{\dfrac{\sin^2 x}{\cos^2 x}+\left(\dfrac{1}{1}\right)\dfrac{\cos^2 x}{\cos^2 x}}{\dfrac{1}{\sin^2 x}-\left(\dfrac{1}{1}\right)\dfrac{\sin^2 x}{\sin^2 x}}$$

$$=\frac{\dfrac{\sin^2 x}{\cos^2 x}+\dfrac{\cos^2 x}{\cos^2 x}}{\dfrac{1}{\sin^2 x}-\dfrac{\sin^2 x}{\sin^2 x}}$$

$$=\frac{\dfrac{\sin^2 x+\cos^2 x}{\cos^2 x}}{\dfrac{1-\sin^2 x}{\sin^2 x}}$$

$$=\frac{\dfrac{1}{\cos^2 x}}{\dfrac{\cos^2 x}{\sin^2 x}}$$

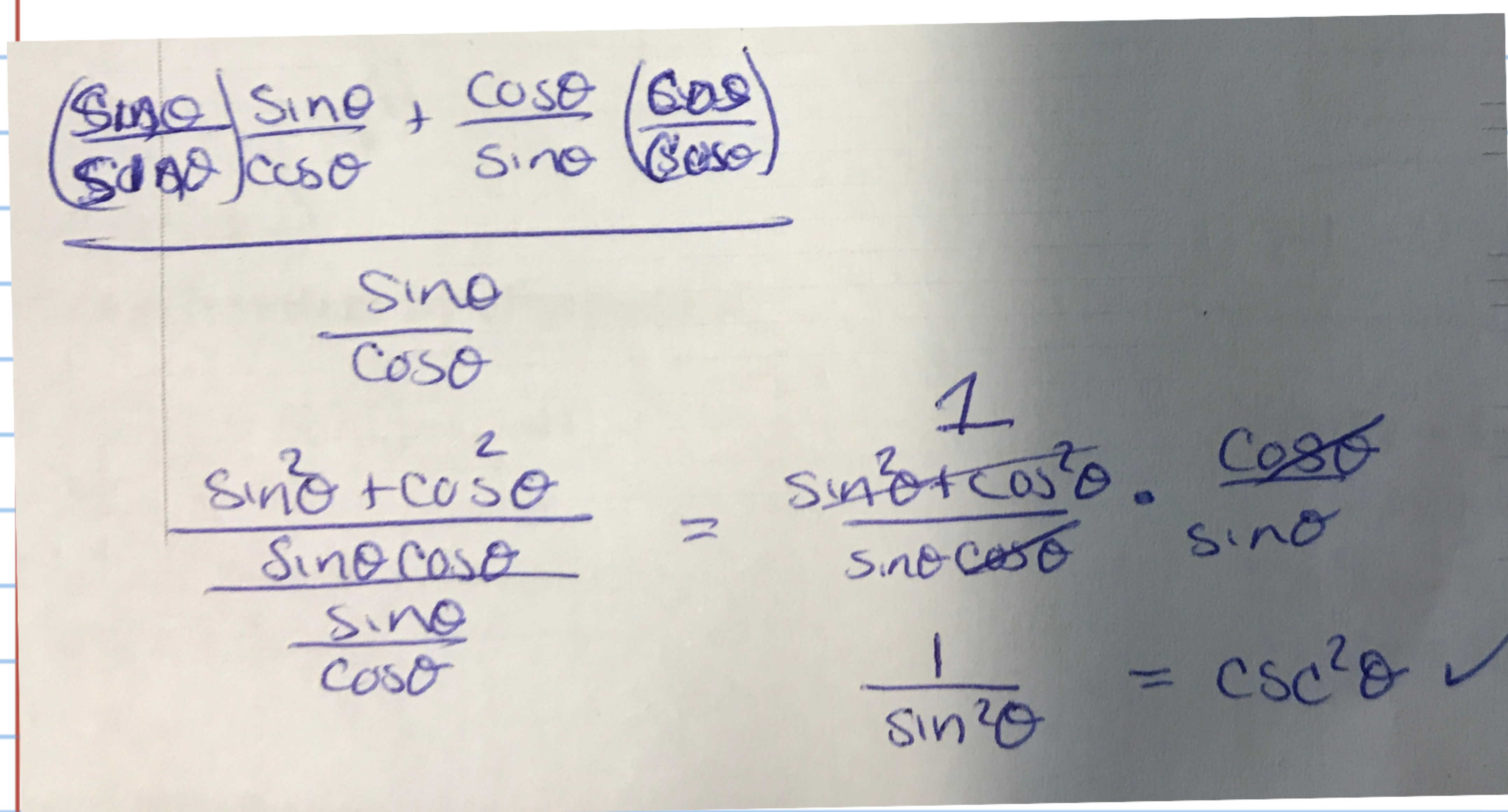
$$=\frac{1}{\cos^2 x}\times\frac{\sin^2 x}{\cos^2 x}$$

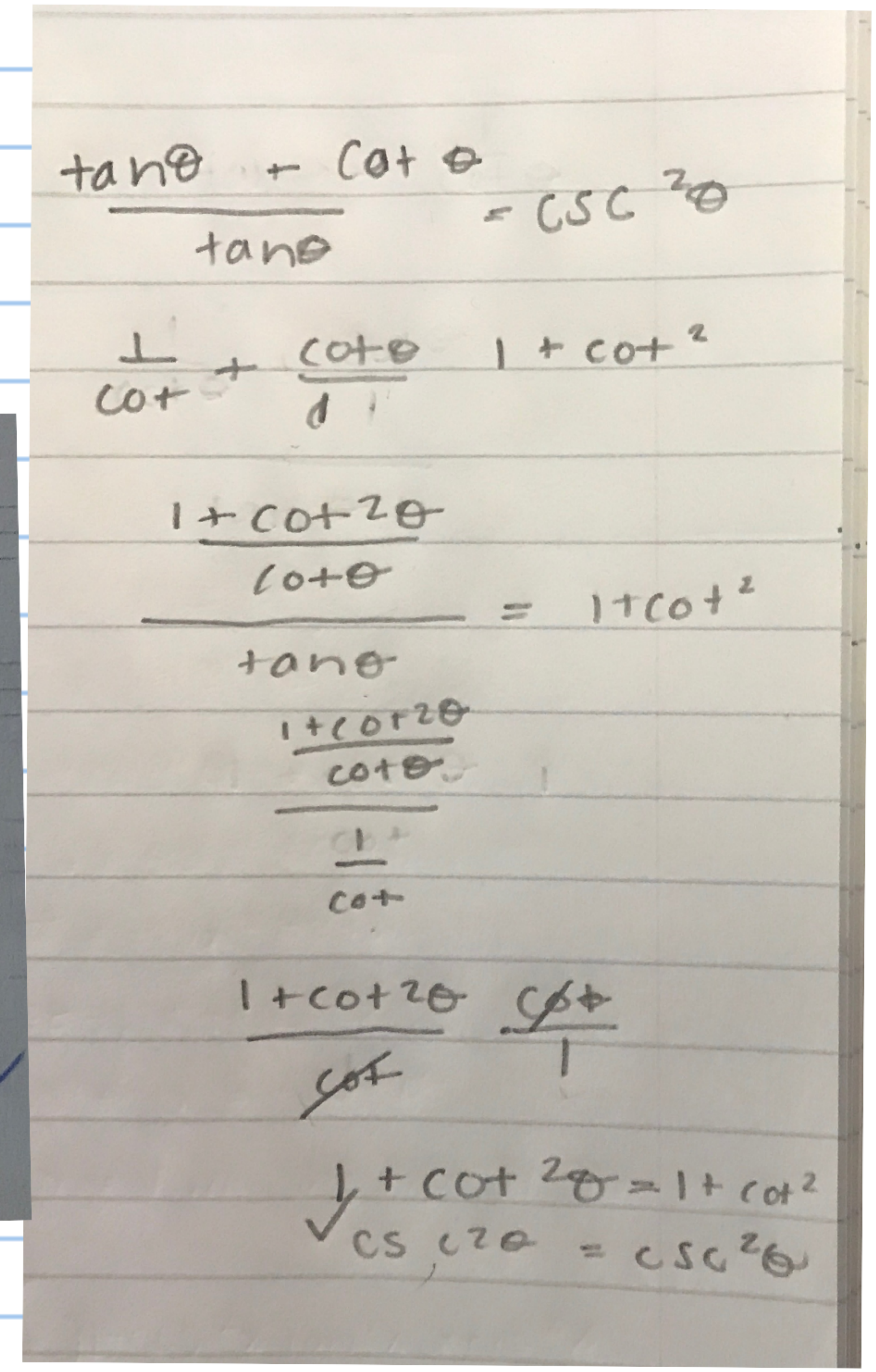
$$=\sec^2 x\tan^2 x$$

Example

Use the trig identities to transform one side of the equation into the other.

$$\frac{\tan\theta + \cot\theta}{\tan\theta} = \csc^2\theta$$





other.

$$\frac{\tan x - \sin x}{\tan x \sin x} = \frac{1 - \cos x}{\sin x}$$

$$\frac{\frac{\sin x}{\cos x} - \frac{\sin x}{1} \cdot \left(\frac{\cos x}{\cos x}\right)}{\frac{\sin x}{\cos x} \cdot \frac{\sin x}{1}}$$

$$\frac{\frac{\sin x - \sin x \cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}}$$

$$\frac{\sin(1 - \cos x)}{\cos x} \cdot \frac{\cos x}{\sin^2 x}$$

$$\frac{1 - \cos x}{\sin x}$$

Example

Use the trig identities to transform one side of the equation into the other.

$$\frac{\cos x - \csc x}{\sin x - \sec x} = \cot x$$

$$= \frac{\cos x - \frac{1}{\sin x}}{\sin x - \frac{1}{\cos x}}$$

$$= \frac{\left(\frac{\cos x}{1}\right)\frac{\sin x}{\sin x} - \frac{1}{\sin x}}{\left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x} - \frac{1}{\cos x}}$$

$$= \frac{\frac{\cos x \sin x}{\sin x} - \frac{1}{\sin x}}{\frac{\cos x \sin x}{\cos x} - \frac{1}{\cos x}}$$

$$= \frac{\frac{\cos x \sin x - 1}{\sin x}}{\frac{\cos x \sin x - 1}{\cos x}}$$

$$= \frac{\cancel{\cos x \sin x - 1}}{\sin x} \times \frac{\cos x}{\cancel{\cos x \sin x - 1}}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

Example

Use the trig identities to transform one side of the equation into the other.

$$\frac{\sin^2 x}{1-\sin x}+\frac{\sin^2 x}{1+\sin x}=2\tan^2 x$$

$$\frac{\sin^2 x}{1-\sin x}+\frac{\sin^2 x}{1+\sin x}$$

$$=\frac{\sin^2 x}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right)+\frac{\sin^2 x}{1+\sin x}\left(\frac{1-\sin x}{1-\sin x}\right)$$

$$=\frac{\sin^2 x+\sin^3 x}{(1-\sin x)(1+\sin x)}+\frac{\sin^2 x-\sin^3 x}{(1+\sin x)(1-\sin x)}$$

$$=\frac{\sin^2 x+\sin^3 x+\sin^2 x-\sin^3 x}{(1-\sin x)(1+\sin x)}$$

$$=\frac{2\sin^2 x}{(1-\sin x)(1+\sin x)}$$

$$=\frac{2\sin^2 x}{(1-\sin^2 x)}$$

$$=\frac{2\sin^2 x}{\cos^2 x}$$

$$=2\tan^2 x$$

Example

Use the trig identities to transform one side of the equation into the other.

$$\frac{\sin x+\tan x}{\cos x+1}=\tan x$$

$$=\frac{\sin x+\frac{\sin x}{\cos x}}{\cos x+1}$$

$$=\frac{\left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\cos x+1}$$

$$=\frac{\frac{\sin x\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\cos x+1}$$

$$=\frac{\frac{\sin x\cos x+\sin x}{\cos x}}{\cos x+1}$$

$$=\frac{\sin x\cos x+\sin x}{\cos x}\times\frac{1}{\cos x+1}$$

$$=\frac{\sin x\cancel{(\cos x+1)}}{\cos x}\times\frac{1}{\cancel{\cos x+1}}$$

$$=\frac{\sin x}{\cos x}$$

$$=\tan x$$