Warm up

1. Use long division to find: $3x^3 - x^2 + 2x - 3$ by $x-2$

2. If $f(x) = 3x^3 - x^2 + 2x - 3$, then what is $f(2)$?

Section 2.3	Real Zeros of Polynomial functions
Objective:	Given a polynomial function students will use long division and synthetic division to divide polynomials to identify its real zeros and use Rational Zero Test to find possible zeros.

Study Problems

Pg 170 # 9, 19, 29, 35, 39, 47, 51- 55 odd, 65, 93

Example 1.

$$
\begin{array}{r|l}
 & 3x^2+5x+12 \\
\hline
x-2 & 3x^3-x^2+2x-3 \\
 & \underline{3x^3-6x^2} \\
 & \quad 5x^2+2x \\
 & \quad \underline{5x^2-10x} \\
 & \qquad 12x-3 \\
 & \qquad \underline{12x-24} \\
 & \qquad\quad 21
\end{array}
$$

$$\boxed{3x^2+5x+12+\frac{21}{x-2}}$$

$$\boxed{(3x^2+5x+12)(x-2)+21}$$

Synthetic division is a nice shortcut for long division of polynomials by divisors of the form x – k.

Let's look at the previous example when done by synthetic division.

$$
\begin{array}{r|rrrr}
2 & 3 & -1 & 2 & -3 \\
 & & 6 & 10 & 24 \\
\hline
 & 3 & 5 & 12 & \vert\, 21
\end{array}
$$

$$\boxed{3x^2+5x+12+\frac{21}{x-2}}$$

Example

Divide $2x^4+4x^3-5x^2+3x-2 \div x^2+2x-3$

$$
\begin{array}{r}
2x^2+1 \\
x^2+2x-3\ \overline{)\ 2x^4+4x^3-5x^2+3x-2} \\
-\cancel{2x^4}\ \cancel{+4x^3}+6x^2 \qquad\qquad \\
\hline
\cancel{x^2}+3x-2 \\
-\cancel{x^2}\ -2x+3 \\
\hline
x+1
\end{array}
$$

$x+1$ ← Remainder

** $\boxed{2x^2+1+\frac{x+1}{x^2+2x-3}}$

OR $(x^2+2x-3)(2x^2+1)+x+1$

Example Evaluate f(-3) using two different methods.

$$f(x)=2x^3-4x^2+1.$$

method 1

$f(-3)=2(-3)^3-4(-3)^2+1$

$=2(-27)-4(9)+1$

$=-54-36+1$

$f(-3)=-89$

method 2

$$
\begin{array}{r|rrrr}
 & x^3 & x^2 & x & c \\
-3 & 2 & -4 & 0 & 1 \\
 & \downarrow & -6 & 30 & -90 \\
\hline
 & 2 & -10 & 30 & -89
\end{array}
$$

opp

$(2x^2-10x+30)+\frac{-89}{x+3}$

$$\frac{2x^4 + 4x^3 - 5x^2 + 3x - 2}{x^2 + 2x - 3} = 2x^2 + 1 + \frac{x + 1}{x^2 + 2x - 3}.$$

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

$$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{Divisor}}\underbrace{q(x)}_{\text{Quotient}} + \underbrace{r(x)}_{\text{Remainder}}$$

solution

$$(2x^2 + 1)(x^2 + 2x - 3) + x + 1$$

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, the remainder is

$r = f(k)$.

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$f(x) = x^3 - x^2 - 14x + 11$, $K = \underline{4}$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & \downarrow & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

Remainde

$f(x) = (x-4)(x^2+3x-2) + 3$

$= (x-k)q(x) + r$