Warm up

Find the zeros of the function.

$f(x) = 3x^2 + x - 10$

a. $x = -\frac{5}{3}$ and $x = -3$

b. $x = -5$ and $x = 2$

c. $x = \frac{5}{3}$ and $x = 2$

d. $x = \frac{5}{3}$ and $x = -2$

Warm up

Find all real zeros.

$f(x) = x^3 - 8x^2 - 20x$

a. The real zeros are $x = 0, x = -10$, and $x = 2$.

b. The real zeros are $x = 0, x = 10$, and $x = -2$.

c. The real zero is $x = 0$.

d. None of the above.

section 2.5 The Fundamental Theorem of Algebra

Objective: Given a function students will find zeros of a polynomial function including complex zeros.

Study Problems
Pg. 187 # 9 - 14, 37, 39, 41, 43, 49, 53

The fundamental theorem of Algebra

-the degree of the polynomial determines the number of zeros.

How many zeros does each polynomial have?

$f(x) = x - 40$ 1 zero

$g(x) = x^2 - 18x - 19.$ 2 zeros

$h(x) = x^2(x+1)$ 3 zeros

$W(x) = x^2(x+1)(x-3)$ 4 zeros

Linear Factorization Theorem

If $f(x)$ is a polynomial of degree n where $n > 0$, f has precisely n linear factors

$$f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$$

where $c_1, c_2, \ldots, c_n$ are complex numbers.

real a imag. $a+bi$ Pure imag. bi

# of zeros is equal to # of factors

Example

Factor $f(x)=x^4-12x^2-13$

a) as a product of factors that are irreducible over the rational numbers.

$f(x) = (x^2 - 13)(x^2+1)$

b) as a product of factors that are irreducible over the real numbers.

$f(x) = (x-\sqrt{13})(x+\sqrt{13})(x^2+1)$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

c) completely over the complex numbers.

$f(x) = (x-\sqrt{13})(x+\sqrt{13})(x-i)(x+i)$

$x = \sqrt{13}, -\sqrt{13}, -i, i$

Example

Write $f(x)= x^3 +6x -7$ as a product of linear factors and list all of its zeros.

$f(x) = (x+7)(x^2-1)$

$7x^2$

$-x$

$6x$

Not Factorable

$\frac{p}{q} \quad \frac{\pm 1, \pm 7}{\pm 1} = \pm 1, \pm 7$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 6 & -7 \\ & \downarrow & 1 & 1 & 7 \\ \hline & 1 & 1 & 7 & 0 \end{array}$$

$f(x) = (x-1)(x^2+x+7)$

quadratic Formula

$x = \frac{-1 \pm \sqrt{1^2-4(1)(7)}}{2(1)}$

$= \frac{-1 \pm \sqrt{1-28}}{2}$

$= \frac{-1 \pm i\sqrt{27}}{2}$ $\quad \sqrt{9}\cdot\sqrt{3}$

$x = \frac{-1 \pm 3i\sqrt{3}}{2}$

$\boxed{\text{Zeros: } 1, \frac{-1+3i\sqrt{3}}{2}, \frac{-1-3i\sqrt{3}}{2}}$

Example

Write $f(x)=x^4-3x^3+x-3$ as a product of linear factors and list all of its zeros.

$= (x^4-3x^3)+(x-3)$

$= x^3(x-3)+1(x-3)$

$= (x-3)(x^3+1)$ $\quad \frac{p}{q} = \frac{\pm 1}{\pm} = \pm 1$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & 1 \\ & \downarrow & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \cancel{2} \end{array}$$

OR

$(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$(a^3-b^3) = (a-b)(a^2+ab+b^2)$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & \downarrow & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array} \checkmark$$

$f(x) = (x-3)(x+1)(x^2-x+1)$ Not Factorable

$x = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$= \frac{1 \pm \sqrt{-3}}{2}$

$x = \frac{1 \pm i\sqrt{3}}{2}$

$\boxed{\text{Zeros: } 3, -1, \frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}}$

Complex Zeros Occur in Conjugate Pairs

Let $f(x)$ be a polynomial function that has *real coefficients*. If $a + bi$, where $b \neq 0$, is a zero of the function, the conjugate $a - bi$ is also a zero of the function.

ex/ If $2i$ is a zero then $-2i$ is also a zero

If $1+\sqrt{5}$ is a zero then $1-\sqrt{5}$ is also a zero.

Example

Find a fourth degree polynomial function with real coefficient that has 0, 1 and i as zeros, *then write in standard form*

$x=0$, $x=1$, $x=i$, $x=-i$

$f(x) = (x+0)(x-1)(x-i)(x+i)$

$= x(x-1)(x-i)(x+i)$

$= (x^2-x)(x-i)(x+i)$ $\quad i^2=-1$

$= (x^2-x)(x^2-\cancel{xi}+\cancel{xi}-\underset{-(-1)}{i^2})$

$= (x^2-x)(x^2+1)$

$= x^4+x^2-x^3-x$

$\boxed{f(x) = x^4-x^3+x^2-x}$

Example

Find all the zeros of $f(x)=x^4-4x^3+12x^2+4x-13$ given that $2+3i$ is a zero, then $2-3i$ is also a zero.

$= [x-(2+3i)][x-(2-3i)]$

$= [(x-2)-3i][(x-2)+3i]$

$= (x-2)^2-3i(+3i)$

$i^2=-1$

$= (x-2)^2-9i^2$

$= x^2-4x+4+9$

$= \underline{x^2-4x+13}$

$$\begin{array}{r|l} & x^2-1 \\ x^2-4x+13 & x^4-4x^3+12x^2+4x-13 \\ & -x^4+4x^3-13x^2 \\ & -x^2+4x-13 \\ & +x^2-4x+13 \\ & 0 \end{array}$$

$= (x^2-4x+13)(x^2-1)$

$(x+1)(x-1)$

$\boxed{\text{Zeros: } 2+3i,\ 2-3i,\ -1,\ 1}$

Think Pair Share

True or False, explain.

1. If $5+6i$ is zero of polynomial $f(x)$ then $-5-6i$ is also a zero of the polynomial $f(x)$.

2. The linear factorization of x^2+15 is $(x-\sqrt{15})(x+\sqrt{15})$.

3. If $7+\sqrt{3}$ is a zero of polynomial $g(x)$ then $7-\sqrt{3}$ is also a zero of the polynomial $g(x)$