Explain the Error Two students used synthetic division to divide $3x^3 - 2x - 8$ by $x - 2$. Determine which solution is correct. Find the error in the other solution.

A.	B.
$\begin{array}{r\|rrrr} 2 & 3 & 0 & -2 & -8 \\ & & 6 & 12 & 20 \\ \hline & 3\ 6 & 10 & 12 \end{array}$	$\begin{array}{r\|rrrr} -2 & 3 & 0 & -2 & -8 \\ & & -6 & 12 & -20 \\ \hline & 3 & -6 & 10 & -28 \end{array}$

Student A is correct. Student B used the incorrect sign of *a*.

The Rational Zero Test on page 166

For a polynomial function

Possible rational zeros = $\dfrac{\text{factors of constant term}}{\text{factors of leading coefficient}}$ $\left(\dfrac{p}{q}\right)$

Find the zeros of the polynomial function

$$f(x) = 3x^3 - 20x^2 + 23x + 10 \quad (P)$$

$$\frac{P}{8} = \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}$$

```
1 | 3  -20   23   10          2 | 3  -20   23   10
  |      3  -17    6            |      6  -28  -10
    3  -17    6   16  11          3  -14   -5    0  ✓ ☺
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$$f(x) = (x-2)(3x^2 - 14x - 5)$$

$$f(x) = (x-2)(3x+1)(x-5)$$

$$\text{Zeros}: \quad 2, -\tfrac{1}{3}, 5$$

Example
List the possible rational zeros of

$$f(x) = 3x^3 - 20x^2 + 23x + 10$$

$$p: \pm 1, \pm 2, \pm 5, \pm 10$$

$$q: \pm 1, \pm 3$$

$$\frac{p}{q}: \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}$$

Example

Determine all the real zeros of f if $(x-1)$ is a factor.

$f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

$= \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2}$

$= \pm 1, \pm 2 \pm 4, \pm 8, \pm \dfrac{1}{2}$

a)

$$\begin{array}{r|rrrrr} 1 & -2 & 13 & -21 & 2 & 8 \\ & \downarrow & -2 & 11 & -10 & -8 \\ \hline & -2 & 11 & -10 & -8 & 0 \end{array}$$

yes, $(x-1)$ is a factor of $f(x)$

b) Find the other solutions

$f(x) = (x-1)(-2x^3 + 11x^2 - 10x - 8)$

$$\begin{array}{r|rrrr} 2 & -2 & 11 & -10 & -8 \\ & \downarrow & -4 & 14 & 8 \\ \hline & -2 & 7 & 4 & 0 \end{array}$$

$f(x) = (x-1)(x-2)(-2x^2 + 7x + 4)$

$f(x) = (x-1)(x-2)(-2x - 1)(x - 4)$

$-1x$
$+8x$
$7x$ ✓

zeros:

$1, 2, -\dfrac{1}{2}, 4$

Example

Determine all the real zeros of f.

$\dfrac{p}{8}$ $\dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2, \pm 3, \pm 6}$

$f(x) = 6x^3 - x^2 - 13x + 8$

$$\begin{array}{r|rrrr} 1 & 6 & -1 & -13 & 8 \\ & \downarrow & 6 & 5 & -8 \\ \hline & 6 & 5 & -8 & 0 \end{array}$$

$f(x) = (x-1)(\overset{a}{6x^2} + \overset{b}{5x} - \overset{c}{8})$

↳ Not Factorble

$x = \dfrac{-5 \pm \sqrt{25 - 4(6)(-8)}}{2(6)}$

$= \dfrac{-5 \pm \sqrt{25 + 192}}{12}$

$x = \dfrac{-5 \pm \sqrt{217}}{12}$

$f(x) = (x-1)\left(x - \left(\dfrac{-5 + \sqrt{217}}{12}\right)\right)\left(x - \left(\dfrac{5 - \sqrt{217}}{12}\right)\right)$

zeros: $1, \dfrac{-5 \pm \sqrt{217}}{12}$