

2.2 Polynomials of higher degree

Real Zeros of Polynomial Functions

If f is a polynomial function and a is a real number, the following statements are equivalent.

1. $x = a$ is a zero of the function f .
2. $x = a$ is a solution of the polynomial equation $f(x) = 0$.
3. $(x - a)$ is a factor of the polynomial $f(x)$.
4. $(a, 0)$ is an x-intercept of the graph of f .

NOTE: Odd powers cross

even powers bounce / touch

Example

Sketch the function by finding the zeros, determine its end behavior, plot points and connect with a curve, state any multiplicities and relative max and mins.

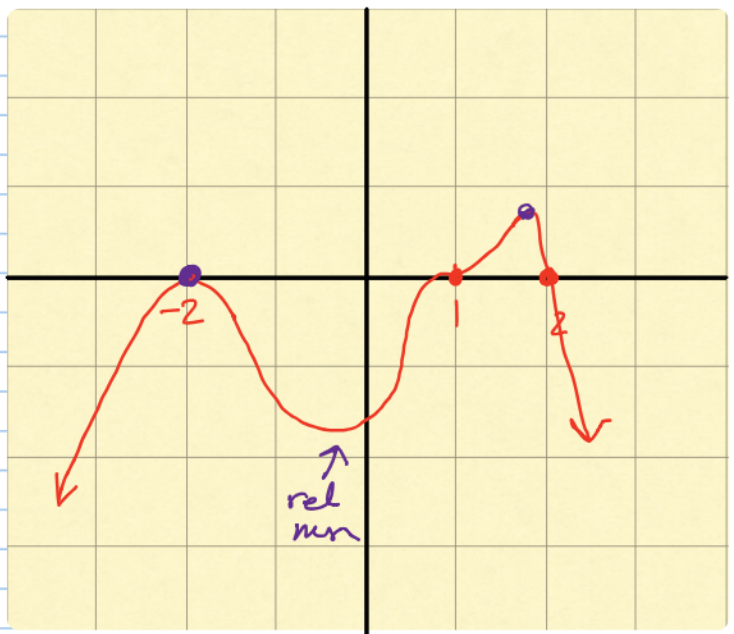
$$f(x) = -2(x-2)(x-1)^3(x+2)^2$$

Degree: 6, L.C.: Negative

| Zeros | Multi. | Behavior on x-axis |
|-------|--------|--------------------|
| 2 | single | Cross / linear / |
| 1 | triple | Cross / Cubic / |
| -2 | double | bounce / quad. / |

rel. max = 2

rel. min = 1



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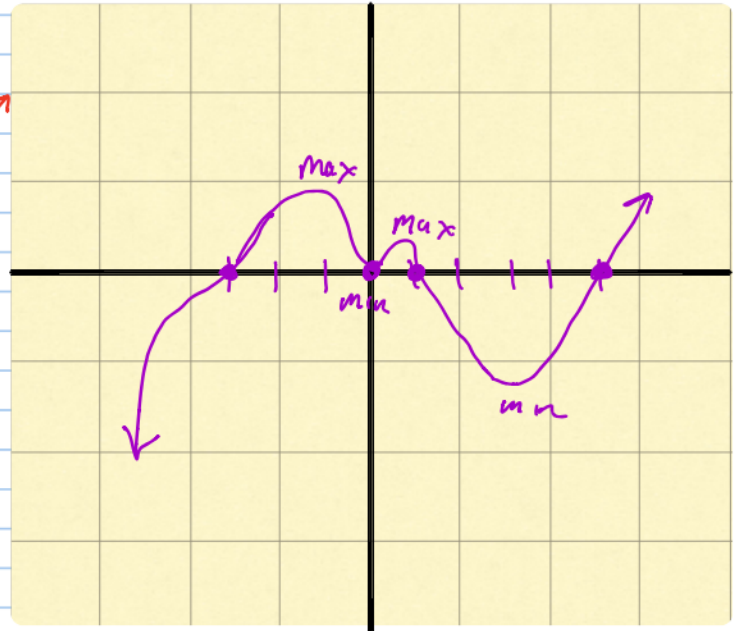
$$f(x) = 3x^2(x-5)(x+3)^3(x-1)$$

Degree: - 7 L.C = +

| Zeros | Mult. | Behavior x -axis |
|-------|--------|--------------------|
| 0 | double | Bounces \cup |
| 5 | single | cross \swarrow |
| -3 | triple | cross \swarrow |
| 1 | single | cross \swarrow |

rel. max: 2

rel. min: 2



Example

Sketch the function by finding the zeros, determine its end behavior, plot points and connect with a curve, state any multiplicities and relative max and mins.

$$f(x) = -2x^4 + 2x^2$$

