Section 2.2	Polynomial functions of higher degree
Objective:	Given a polynomial function students will be able to sketch its graph using facts learned in math 3, such as leading coefficient test, fundamental theorem of algebra, multiplicity, etc.

Study Problems

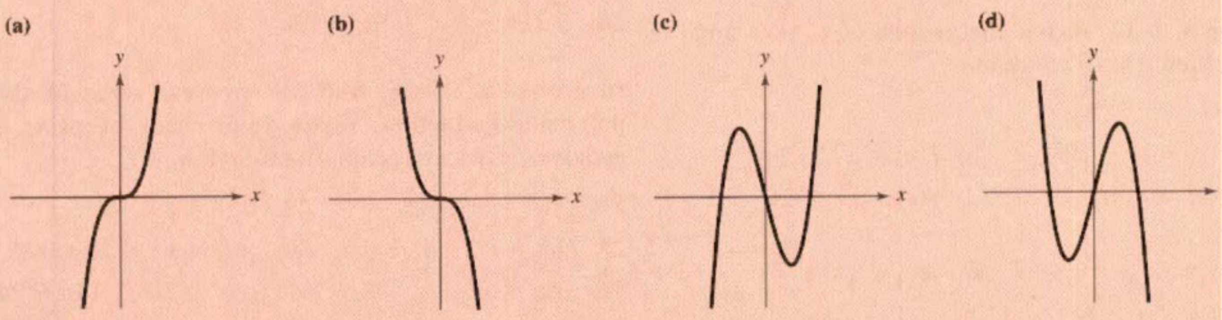
Page 156 #10-11, 23, 25, 33-35, 47, 51, 59, 63, 73, 77

Warm Up

Page 155

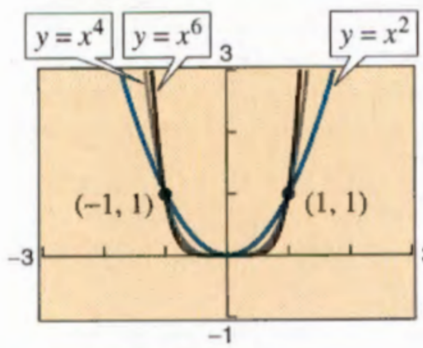
Writing About Math *The Graphs of Cubic Polynomials*

The graphs of cubic polynomials can be categorized according to the four basic shapes below. Match the graph of each function with one of the basic shapes and write a short paragraph describing how you reached your conclusion. Is it possible for a polynomial of odd degree to have no real zeros? Explain.

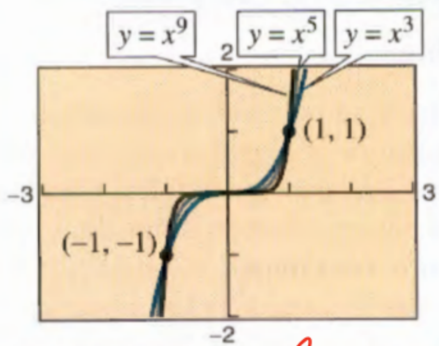


1. $f(x) = -x^3$
2. $f(x) = -x^3 + 4x$
3. $f(x) = x^3$
4. $f(x) = x^3 - 4x$

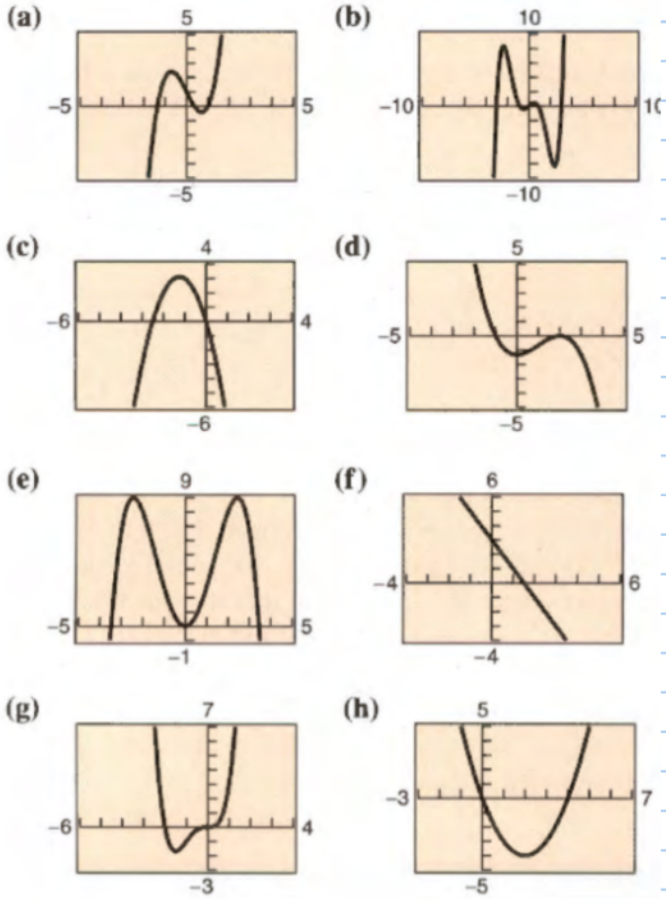
Continuous The graph of a polynomial function has no breaks, holes, or gaps. Its graph also is smooth with rounded turns.



If n is even, the graph of $y = x^n$ touches the axis at the x-intercept.



If n is odd, the graph of $y = x^n$ crosses the axis at the x-intercept.



Match the graph with the function.

1. $f(x) = -2x + 3$

3. $f(x) = -2x^2 - 5x$

5. $f(x) = -\frac{1}{4}x^4 + 3x^2$

7. $f(x) = x^4 + 2x^3$

2. $f(x) = x^2 - 4x$

4. $f(x) = 2x^3 - 3x + 1$

6. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$

8. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$

2.2 Polynimials of higher degree

Example

Describe the right-hand and left- hand behavior of the graph of each of the following functions. Justify your response

a) $f(x) = -x^4 + 7x^3 - 14x - 9$

b) $g(x) = 5x^5 + 2x^3 - 14x^2 + 6$

c) $h(x) = -x^5 + 3x^4 - x$

a) Since the L.C. Negative and degree is even, then both sides are down

Left ↙ ↓ right

$x \to \infty$, $f(x) = -\infty$

$x \to -\infty$, $f(x) = -\infty$

b) ↙ ↗

Since the L.C is positive & degree is odd, then Falls on the left and rises on the right.

$x \to \infty$, $f(x) = \infty$

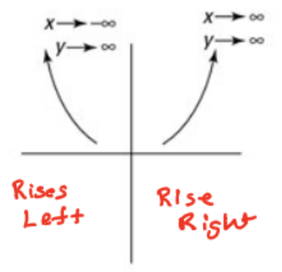
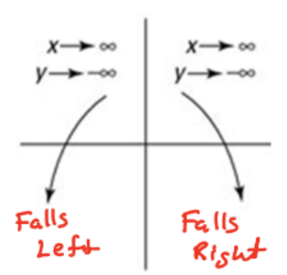
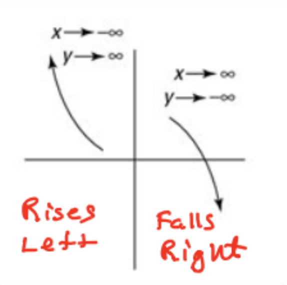
$x \to -\infty$, $f(x) = -\infty$

c) ↗ ↓

Since the L.C is Negative & degree is odd, then rises on the left and Falls on the right.

$x \to \infty$, $f(x) = -\infty$

$x \to -\infty$, $f(x) = +\infty$

	Positive Coefficient	Negative Coefficient
Even Degree x^2 x^4 x^6	 $x \to -\infty$, $y \to \infty$; $x \to \infty$, $y \to \infty$ Rises Left Rise Right	 $x \to \infty$, $y \to -\infty$; $x \to \infty$, $y \to -\infty$ Falls Left Falls Right
Odd Degree x^3 x^5 x^7	 $x \to -\infty$, $y \to -\infty$; $x \to \infty$, $y \to \infty$ Falls Left Rises Right	 $x \to -\infty$, $y \to \infty$; $x \to \infty$, $y \to -\infty$ Rises Left Falls Right

Real Zeros of Polynomial Functions

If f is a polynomial function and a is a real number, the following statements are equivalent.

1. x = a is a zero of the function *f*.
2. x = a is a solution of the polynomial equation *f(x)* = 0.
3. (x - a) is a factor of the polynomial *f(x)*.
4. (a,0) is an x-intercept of the graph of *f*.

NOTE: Odd powers cross
even powers bounce/touch

Example

Sketch the function by finding the zeros, determine its end behavior, plot points and connect with a curve, state any multiplicities and relative max and mins.

$f(x) = -2(x-2)(x-1)^3(x+2)^2$