Simplifying Radical Expressions

- If there is no index #, it is understood to be 2
- When simplifying radicals use perfect squares, cubes, etc.
- Use factor to break a number into its prime factors
- Apply the properties of radicals and exponents

$\sqrt[2]{\ }$ $\sqrt[3]{\ }$ $\sqrt[4]{\ }$

What numbers are perfect squares?

1 • 1 = **1**

2 • 2 = **4**

3 • 3 = **9**

$4^2 =$ 4 • 4 = **16**

5 • 5 = **25**

6 • 6 = **36**

49, 64, 81, 100, 121, 144, ...

Square roots are inverses of squaring a number. Example: $4^2 = 16$, so $\sqrt{16} = 4$

When you are trying to find a square root, ask yourself, "what number when multiplied by itself (squared) would equal this number"? In the above example, "what number when multiplied by itself (squared) would be 16?" 4 because 4·4=16. $\sqrt{16}$

Cube roots are the inverses of cubing a number. Example $4^3 = 64$, so $\sqrt[3]{64} = 4$

When you are trying to find a cube root, ask yourself, "what number when multiplied by itself 3 times (cubed) would be this number"? In the above example, "what number when multiplied by itself 3 times (cubed) would be this number"? 4 because 4·4·4=64

$8 = 2\cdot2\cdot2 = 2^3$ $\quad \sqrt[3]{8} = \sqrt[3]{2^3} = 2$

$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$ $\quad 27 = 3\cdot3\cdot3 = 3^3$

3

Perfect Squares and Cubes are equal to whole number values.

Perfect Square Roots		Perfect Cube Roots	
$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^2 = 49$	$\sqrt{49} = 7$	Keep in mind that all square roots have both positive and negative answers. For example, $\sqrt{4}$ can be 2 or -2 since -2 · -2 = 4, but the positive root is called the principal root. If the square root symbol is used, give the principal (positive) square root.	
$8^2 = 64$	$\sqrt{64} = 8$		
$9^2 = 81$	$\sqrt{81} = 9$		
$10^2 = 100$	$\sqrt{100} = 10$		
$11^2 = 121$	$\sqrt{121} = 11$		
$12^2 = 144$	$\sqrt{144} = 12$		

Simplify

$\sqrt{4} = 2$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{100} = 10$

$\sqrt{144} = 12$

Product Property for Radicals

$50 = 25 \cdot 2$

$$\sqrt{50} = \sqrt{25} \bullet \sqrt{2} = 5\sqrt{2}$$

- A radical has been simplified when its radicand contains no perfect square factors.
- Test to see if it can be divided by 4, then 9, then 25, then 49, etc.
- Sometimes factoring the radicand using the "tree" is helpful.

$$\sqrt{x^{14}} = x^7$$

3 or cake

$\sqrt[3]{x^9} = x^3$ $\quad \sqrt{x^{24}} = x^{12}$

$\boxed{\sqrt{x^{14}} = x^7}$

Look at these examples and try to find the pattern…

$\sqrt{x^1} = \sqrt{x}$

$\sqrt{x^2} = x$

$\sqrt{x^3} = x\sqrt{x}$

$\sqrt{x^4} = x^2$

$\sqrt{x^5} = x^2\sqrt{x}$

$\sqrt{x^6} = x^3$

What is the answer to $\sqrt{x^7}$?

$\sqrt{x^7} = \sqrt{x^6 x^1} = x^3\sqrt{x}$

As a general rule, divide the exponent by two. The remainder stays in the radical.

Steps

1. Try to divide the radicand into a perfect square for numbers
2. If there is an exponent make it even by using rules of exponents
3. Separate the factors to its own square root
4. Simplify

1 Explore

Simplify

$$\sqrt{288}$$

2 Explore

Simplify

A $\sqrt{18}$

$\sqrt{9\cdot 2}$

$3\sqrt{2}$

9 | 18
2 | 2
1

B $\sqrt{75}$

$\sqrt{25\cdot 3}$

$5\sqrt{3}$

25 | 75
3 | 3
1

C $\sqrt{24}$

$\sqrt{4\cdot 3\cdot 2}$

$2\sqrt{6}$

4 | 24
3 | 6
2 | 2
1

D $\sqrt{72}$

$\sqrt{9\cdot 4\cdot 2}$

$3\cdot 2\sqrt{2}$

$6\sqrt{2}$

9 | 72
4 | 8
2 | 2
1