Introduction to Limits

Objective: Given a limit problem students will be able to evaluate the limit of the function.

# Techniques for Evaluating Limits

1. Substitution
2. Factoring
3. The Conjugate Method

Substitution

$$\lim_{x \to 0} \frac{x^2 - 1}{2x + 3}$$

$$\lim_{x \to 0} \frac{x^2 - 1}{2x + 3}$$
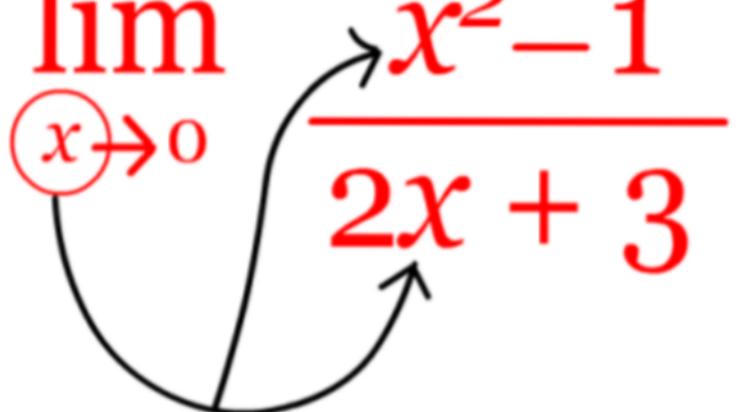
$$\lim_{x \to 0} \frac{0^2 - 1}{2(0) + 3} = \frac{-1}{3}$$

Find the following:

1. $\lim\limits_{x \to 4}(5x - 2)$ $= 18$

2. $\lim\limits_{x \to 9}(\sqrt{x} - 8)$ $= -5$

3. $\lim\limits_{x \to 7} 20$ $= 20$

4. $\lim\limits_{x \to 3} 6x^2/(x - 2)$ $= 54$

5. $\lim\limits_{x \to \pi}(\cos 2x)$

1

6. $\lim\limits_{x \to 0} \dfrac{1}{\cos x \tan x}$

undefined

Now that you should be clear on how to determine limits from the graph, let's move on to trying to determine the limit given the algebraic representation. Consider the function,

$$g(x) = \begin{cases} -x, & x < -2 \\ \dfrac{1}{2}x^2 - 1, & -2 \le x < 2 \\ -x + 3, & x > 2 \end{cases}$$

Again, let's start off by asking a couple of algebra questions. Determine:

$$g(-5) = -(-5) = 5 \qquad g(-2) = \tfrac{1}{2}(-2)^2 - 1$$
$$= 1$$
$$g(0) = \tfrac{1}{2}(0)^2 - 1 = -1 \qquad g(2) = \text{undefined}$$
$$g(3) = -3 + 3 =$$

$$g(x) = \begin{cases} -x, & x < -2 \\ \dfrac{1}{2}x^2 - 1, & -2 \le x < 2 \\ -x + 3, & x > 2 \end{cases}$$

find the following limits:

$$\lim_{x \to -3} g(x) = ? \quad -(-3) = 3$$

$$\lim_{x \to -1} g(x) = ? \quad = -\dfrac{1}{2}$$

$$\lim_{x \to 4} g(x) = ? \quad = -1$$

Now that you should be clear on how to determine limits from the graph, let's move on to trying to determine the limit given the algebraic representation. Consider the function,

$$g(x) = \begin{cases} -x, & x < -2 \\ \dfrac{1}{2}x^2 - 1, & -2 \le x < 2 \\ -x + 3, & x > 2 \end{cases}$$

$$\lim_{x \to -2^-} g(x) = 2$$

$$\lim_{x \to -2^+} g(x) = 1$$

$$\lim_{x \to -2} g(x) = DNE$$
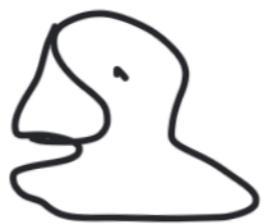
Find the following limits for the piecewise function: $f(x) = \begin{cases} x+1, & x < 2 \\ x^2 - 2, & 2 < x < 4 \\ \sqrt{x+5}, & x \geq 4 \end{cases}$

13. $\lim\limits_{x \to 1^+} f(x) = 2$

14. $\lim\limits_{x \to 1^-} f(x) = 2$

15. $\lim\limits_{x \to 1} f(x) = 2$

16. $f(1) = 2$

17. $\lim\limits_{x \to 2^+} f(x)\ 2$

18. $\lim\limits_{x \to 2^-} f(x) = 3$

19. $\lim\limits_{x \to 2} f(x) = $ DNE

20. $f(2) = $ DNE undefine

21. $\lim\limits_{x \to 3^+} f(x) = 7$

22. $\lim\limits_{x \to 3^-} f(x) = 7$

23. $\lim\limits_{x \to 3} f(x) = 7$

24. $f(3) = 7$

25. $\lim\limits_{x \to 4^+} f(x) = 3$

26. $\lim\limits_{x \to 4^-} f(x) = 14$

27. $\lim\limits_{x \to 4} f(x) = $ DNE

28. $f(4) = 3$