

## Theorem Sum and Difference Formulas for Sines

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

## Example

Find the exact value of

$$\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$$

$$\sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3}$$

$$\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

## Example

Use the sum trig formula to solve the equation in the interval  $[0, 2\pi]$ .

$$\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 1$$

$$\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} + \sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} = -1$$

$$2 \sin x \cos \frac{\pi}{4} = -1$$

$$2 \sin x \left(\frac{\sqrt{2}}{2}\right) = -1$$

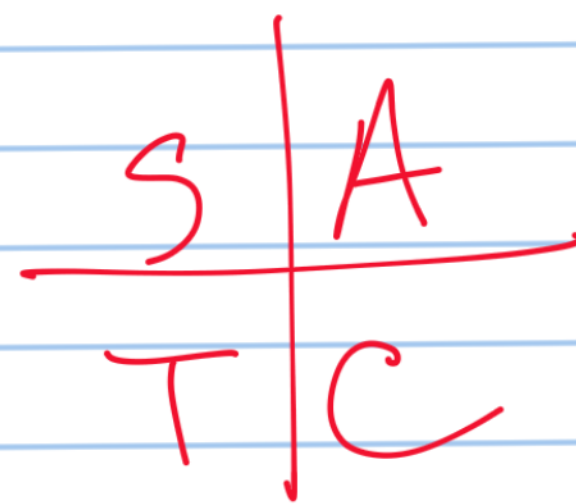
$$\sin x (\sqrt{2}) = -1$$

$$\sin x = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$



$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$



## Theorem Sum and Difference Formulas for Tangents

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

## Example

Simplify using the tangent sum formula

$$\tan(x + 3\pi)$$

$$\frac{\tan x + \tan 3\pi}{1 - \tan x \tan 3\pi}$$

$$\frac{\tan x + 0}{1 - \tan x(0)}$$

$$\tan x$$

$$3\pi (-1, 0)$$

$$\frac{\sin 3\pi}{\cos 3\pi} = \frac{0}{-1}$$

$$= 0$$