Section 5.4 | Sum and Difference Trig Formulas

Objective: Given the formulas for sum and difference for trig function students will be able to find exact solutions to trig functions not on the unit circle.

Study Problems

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Example

Use graphing calculator to determine which of the following is an identity.

1. $\cos(5+x)=\cos 5\cos x-\sin 5\sin x$

2. $\cos(5+x)=\cos 5+\cos x$

$\cos(5-x)=\cos 5\cos x+\sin 5\sin x$

Theorem Sum and Difference Formulas for Cosines

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

Example

Find the exact value of

$$\cos(105^o) = \cos(60^\circ + 45^\circ)$$

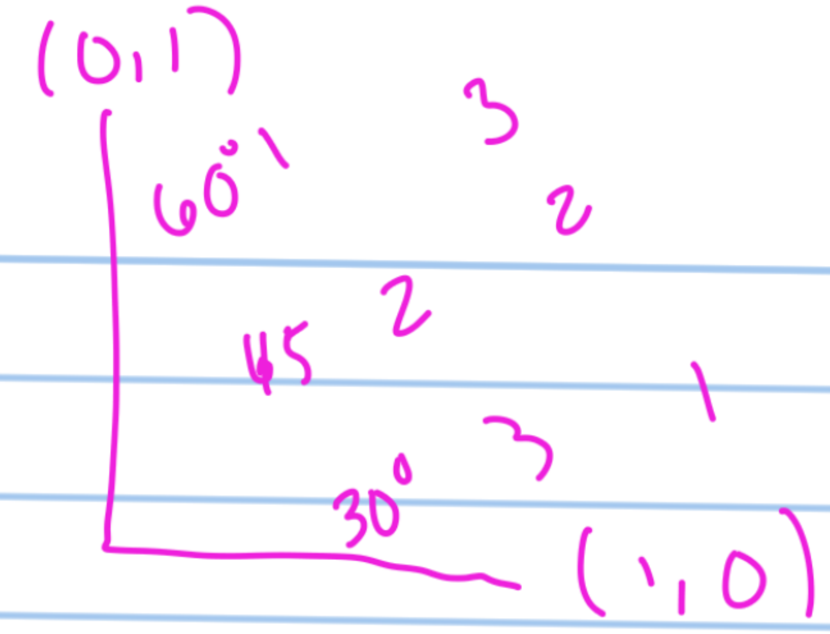
$$= \cos 60 \cos 45 - \sin 60 \sin 45$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

☆ exact value



Example

Find the exact value of

$$\cos\frac{7\pi}{12}$$

$$\cos\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right)$$

$$\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$$

$$\cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$$

$$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

How can we write this function to use the sum or difference formula to find the exact value of the following expression?

Example

Use the sum trig formula to solve the equation in the interval [0, 2π].

$$\cos\left(x+\frac{\pi}{6}\right)-\cos\left(x-\frac{\pi}{6}\right)=1$$

$$\cos x\cos\frac{\pi}{6}-\sin x\sin\frac{\pi}{6}-\left(\cos x\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}\right)=1$$

$$\cancel{\cos x\cos\frac{\pi}{6}}-\sin x\sin\frac{\pi}{6}\cancel{-\cos x\cos\frac{\pi}{6}}-\sin x\sin\frac{\pi}{6}=1$$

$$-2\sin x\sin\frac{\pi}{6}=1$$

$$-\cancel{2}\sin x\left(\frac{1}{\cancel{2}}\right)=1$$

$$-\sin x=1$$

$$\sin x=-1$$

$$\boxed{\frac{3\pi}{2}}$$

Example

Use graphing calculator to determine which of the following is an identity.

1. $\sin(5+x)=\sin 5\cos x+\cos 5\sin x$

True

2. $\sin(5+x)=\sin 5+\sin x$

False

$\sin(5-x)=\sin 5-\sin x$

False

Theorem Sum and Difference Formulas for Sines

$$\boxed{\begin{aligned}\sin(\alpha+\beta)&=\sin\alpha\cos\beta+\cos\alpha\sin\beta\\ \sin(\alpha-\beta)&=\sin\alpha\cos\beta-\cos\alpha\sin\beta\end{aligned}}$$

Example

Find the exact value of

$$\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$$

$$\sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3}$$

$$\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$