Section 5.1 & 5.2

Part 1

Using Trigonometric Identities

Objective: Given an **equation** students will be able to prove/verify the equation is true by using the ***fundamental*** trig identities.

Study problems

Trigonometry II Part 1wks

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities $x^2 + y^2 = 1^2$

$\sin^2\theta + \cos^2\theta = 1$ → $\sin^2 = 1 - \cos^2 x$ → $\cos^2 x = 1 - \sin^2 x$

$1 + \tan^2\theta = \sec^2\theta$ → $\tan^2\theta = \sec^2\theta - 1$, $1 = \sec^2\theta - \tan^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

$$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$$

Even and Odd Identities

$$\sin(-\theta) = -\sin\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\cot(-\theta) = -\cot\theta$$

odd functions reflect across the Origin

$$\sec(-\theta) = \sec\theta$$

$$\cos(-\theta) = \cos\theta$$

even function reflect across the y-axis.

Type I: Identities with multiplication & division:

In these proofs, you will need to convert everything to sine and cosine, then use fraction multiplication & division to simplify.

$$\tan x \cot x = 1$$

$$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}$$

$$1 = 1 \checkmark$$

Fraction Review

Multiplying Fractions:

To multiply fractions, simply multiply the numerators together, and the denominators together.

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos^2 x}$$

$$\frac{\frac{\sin x}{\cos x}}{\cos x} = \frac{\sin x}{\cos^2 x}$$

Fraction Review

Canceling:

When multiplying fractions you will frequently find factors that can be cancelled. You can cancel something on top with something identical on the bottom.

$$\frac{\cos x}{\sin x} \times \frac{\sin^2 x}{\cos x}$$

$$= \frac{\cancel{\cos x}}{\cancel{\sin x}} \times \frac{\sin^{\cancel{2}} x}{\cancel{\cos x}}$$

$$= \sin x$$

Dividing Fractions:

When dividing fractions, rewrite the top fraction, then multiply by the reciprocal of the bottom fraction.

$$\frac{\frac{\sin x}{\cos x}}{\cos x} = \frac{\sin x}{\cos x} \times \frac{1}{\cos x} = \frac{\sin x}{\cos^2 x}$$

Example

Use the trig identities to transform one side of the ~~equation~~ identity into the other.

$$\cos\theta \sec\theta = 1$$

$\cos\theta \left(\frac{1}{\cos\theta}\right)$

$\frac{1}{\sec\theta}$ $\sec\theta$

$1 = 1$ ✓

Example

Use the trig identities to transform one side of the equation into the other.

$$\sin\theta \sec\theta = \tan\theta$$

$\sin\theta\left(\frac{1}{\cos\theta}\right) =$

$\frac{\sin\theta}{\cos\theta}$

$\tan\theta = \tan\theta$ ✓

Example

Use the trig identities to transform one side of the equation into the other.

$$\csc\theta\tan\theta = \sec\theta$$

$$\frac{1}{\sin\theta}\cdot\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$\frac{1}{\cos\theta}$$

$$\sec\theta = \sec\theta \checkmark$$

Example

Use the trig identities to transform one side of the equation into the other.

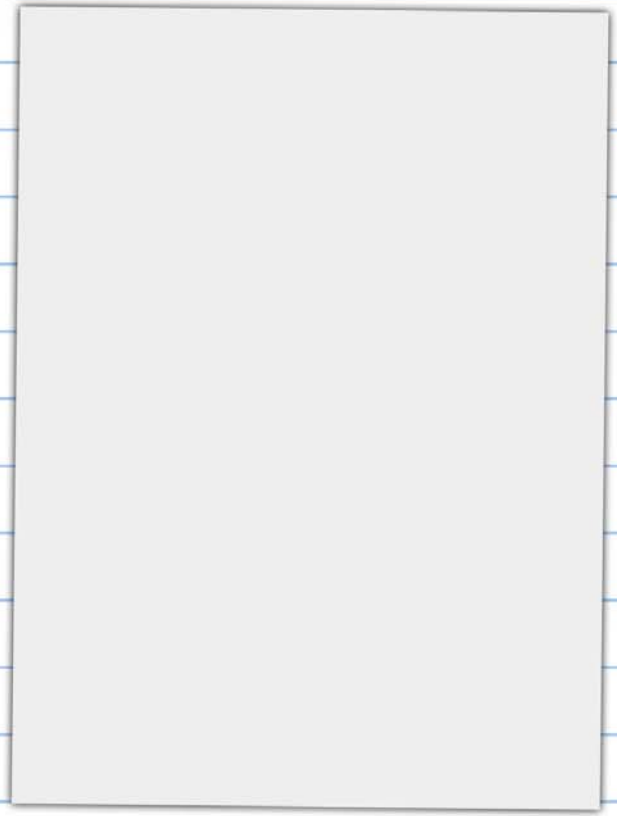
$$\frac{\tan x \cos x}{\sin x} = 1$$

$$\frac{\tan x \cos x}{\sin x}$$

$$= \frac{\left(\frac{\sin x}{\cancel{\cos x}}\right)\cancel{\cos x}}{\sin x}$$

$$= \frac{\sin x}{\sin x}$$

$$= 1$$



other.

$$\frac{\csc x}{\cot x} = \sec x$$

$$\frac{\csc x}{\cot x}$$

$$= \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$$

$$= \frac{1}{\sin x} \times \frac{\sin x}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$