Section 4.7

Part 3

Composition of Inverse Trig Functions

Objective: Given a composition function involving an inverse trig function students will use their properties to solve.

Study Problems

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Inverse Properties of Trigonometric Functions

If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

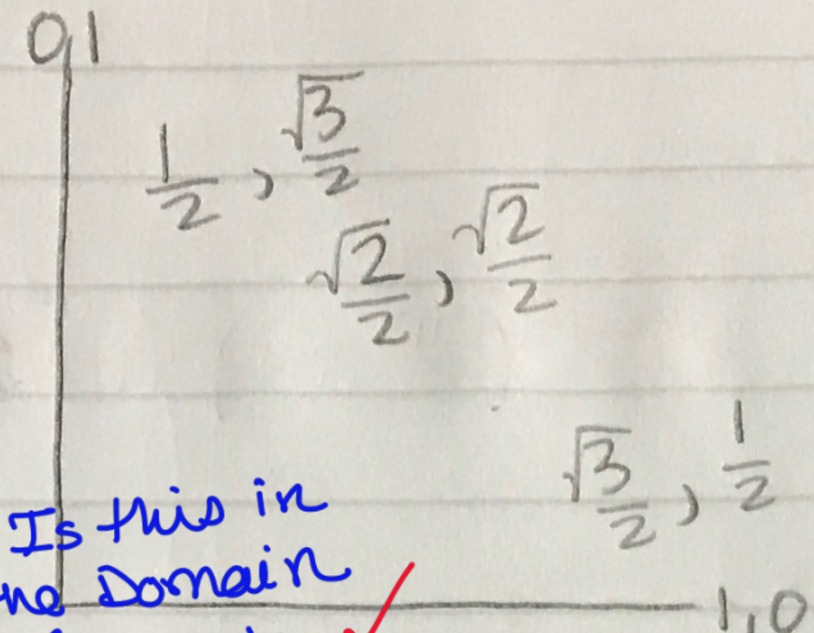
$$\sin(\arcsin x) = x \qquad \text{and} \qquad \arcsin(\sin y) = y.$$

If $-1 \le x \le 1$ and $0 \le y \le \pi$, then

$$\cos(\arccos x) = x \qquad \text{and} \qquad \arccos(\cos y) = y.$$

If x is a real number and $-\pi/2 < y < \pi/2$, then

$$\tan(\arctan x) = x \qquad \text{and} \qquad \arctan(\tan y) = y.$$



* never gonna be in the 3rd quadrant

a. $\sin(\arcsin \frac{1}{2})$

Is this in the Domain of arcsin. ✓

Then find the answer in the Range of arcsin $(-\frac{\pi}{2}, \frac{\pi}{2})$

$\sin(\pi/6)$

$= \boxed{\frac{1}{2}}$

b. $\arctan(\tan \frac{\pi}{4})$

Is this in Range of af arctan? ✓ $\forall \mathbb{R}$ #'s

$\arctan(1)$

Find Answer in Range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\boxed{\frac{\pi}{4}}$

c. $\cos(\cos^{-1} \frac{\sqrt{3}}{2})$

Is that in the Domain of $\cos^{-1}$? ✓

Then Find I's Range value.

$\cos(\pi/6)$

$\boxed{\frac{\sqrt{3}}{2}}$

Evaluate the composition of the trig function

Inverse Properties of Trigonometric Functions

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$\cos(\arccos x) = x$ and $\arccos(\cos y) = y$.

If x is a real number and $-\pi/2 < y < \pi/2$, then

$\tan(\arctan x) = x$ and $\arctan(\tan y) = y$.

Is this in the Range of $\cos^{-1}$. ✓

$a. \ \cancel{\cos^{-1}}(\cancel{\cos}\pi)$

Then we Can Cancel.

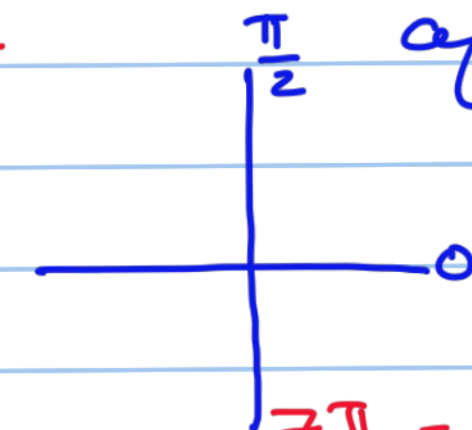
$= \pi$

$b. \ \arccos\left(\cos\frac{7\pi}{2}\right)$

$=$ Undefined

Is $\frac{7\pi}{2}$ in the Range of arccos?

✓



$\frac{7\pi}{2} = 0$

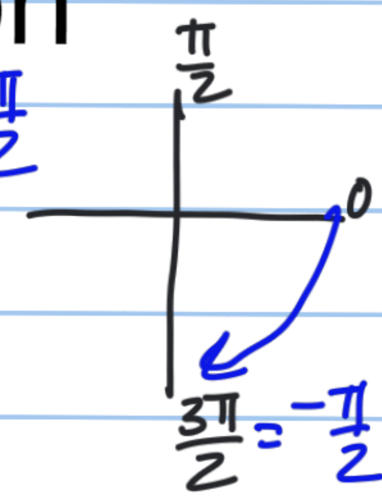
In what interval does the angle for cosine need to be so that $\underline{\arccos(\cos x)}=x$?

It has to be in the Range $[0, \pi]$ to Cancel out the arccos w/ cos x.

Evaluate the composition of the trig function

$a.\ \arcsin\left(\sin\frac{3\pi}{2}\right)$ $-\frac{\pi}{2}$

$\boxed{-\frac{\pi}{2}}$



Is this in the Range of

$b.\ \arcsin(\sin .5\pi)$ $\frac{\pi}{2}$

$\boxed{\frac{\pi}{2}}$

Inverse Properties of Trigonometric Functions

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If $-1 \le x \le 1$ and $0 \le y \le \pi$, then

$\cos(\arccos x) = x$ and $\arccos(\cos y) = y$.

If x is a real number and $-\pi/2 < y < \pi/2$, then

$\tan(\arctan x) = x$ and $\arctan(\tan y) = y$.

What do we need to consider when evaluating a inverse composition function?

Their Domain & Range & if they are Coterminal ∡'s/Radians.

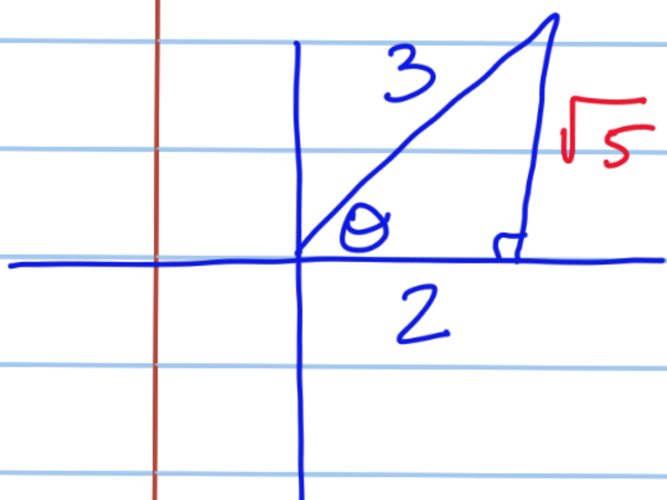
Sohcahtoa

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

since $2/3$ is not on unit ⊙. we need a Δ

Write each of the following as a algebraic expression.

$a.\ \tan(\arccos\frac{2}{3})$



$2^2+b^2=3^2$

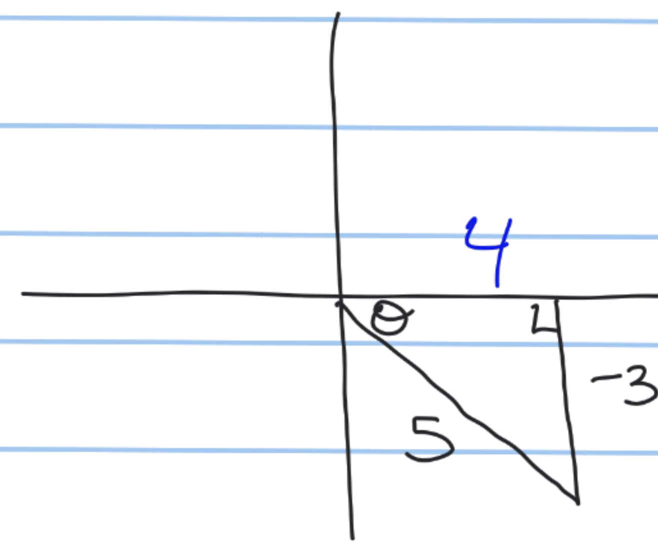
$4+b^2=9$

$b^2=5$

$b=\sqrt{5}$

$\tan\theta=\frac{o}{a}=\boxed{\frac{\sqrt{5}}{2}}$

$b.\ \cos\left(\sin^{-1}\left(\frac{-3}{5}\right)\right)$



$a^2+(-3)^2=5^2$

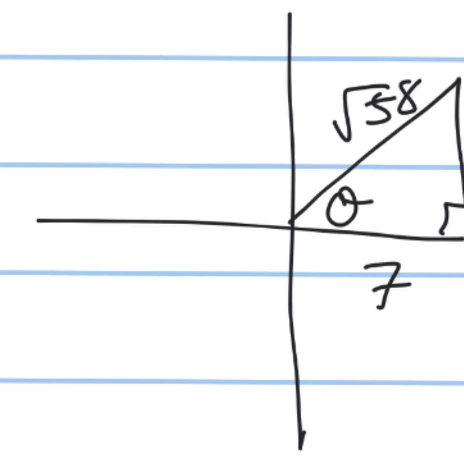
$a^2+9=25$

$a^2=16$

$a=4$

$\cos\theta=\frac{a}{h}=\boxed{\frac{4}{5}}$

$c.\ \sec(\tan^{-1}(\frac{3}{7}))$



$a^2=3^2+7^2$

$a^2=9+49$

$a^2=58$

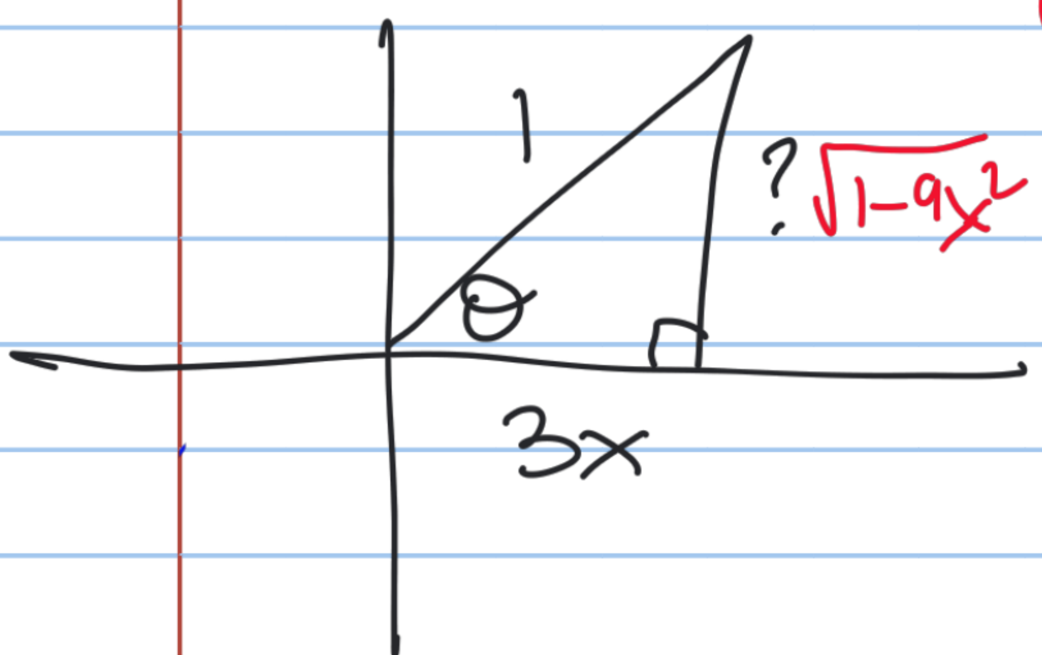
$a=\sqrt{58}$

$\sec\theta=\frac{h}{a}=\boxed{\frac{\sqrt{58}}{7}}$

Sohcahtoa

Write each of the following as a algebraic expression.

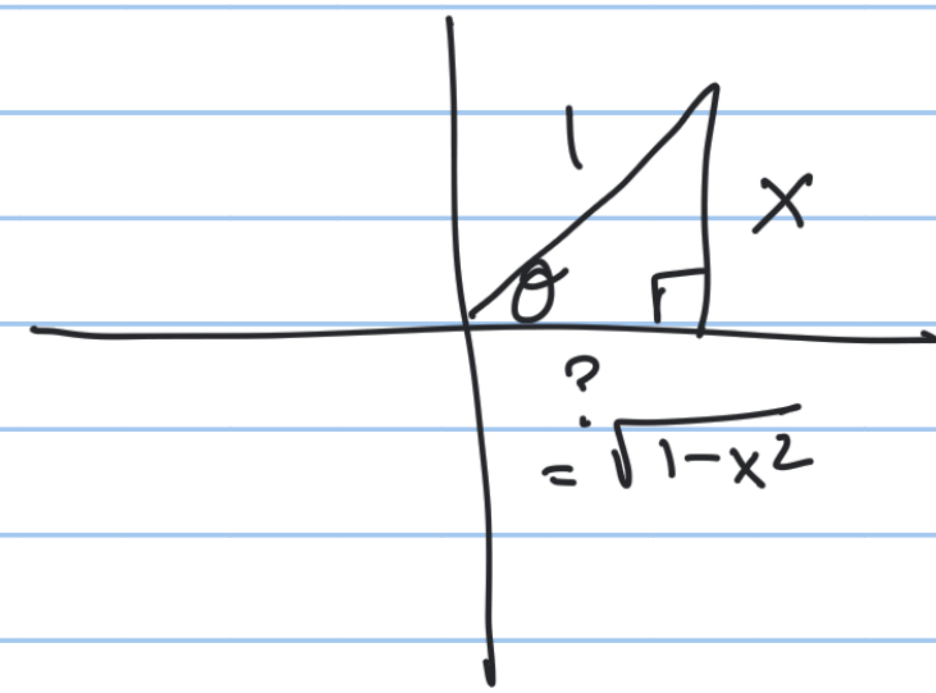
$a.\ \sin\left(\arccos \frac{3x}{1}\right)$



$$(3x)^2 + b^2 = 1^2$$
$$9x^2 + b^2 = 1$$
$$b^2 = 1 - 9x^2$$
$$b = \sqrt{1-9x^2}$$

$$\sin\theta = \frac{o}{h} = \boxed{\frac{\sqrt{1-9x^2}}{1}}$$

$b.\ \cos\left(\sin^{-1} \frac{x}{1}\right)$



$$a^2 + x^2 = 1^2$$
$$a^2 = 1 - x^2$$
$$a = \sqrt{1-x^2}$$

$$\cos\theta = \frac{a}{h} = \boxed{\sqrt{1-x^2}}$$