Section 3.3 Properties of Logs

Objective Given the properties of logarithms, students will be able to write equivalent log expressions.

Study Problems

Pg 244 # 30-65*5, (73-93 odd)

True or false? Explain your answer.

1. The equation $a^3=2$ is equivalent to $\log_a 2=3$.

2. $5^{\log_5 125}=125$

3. If the order pair (a,b) satisfies $y=8^x$, then the same order pair (a,b) satisfies $y=\log_8 x$.

4. log -10=1

5. If $f(x)=a^x$ for $a>0$ and a is not 1 then $f^{-1}(x)=\log_a x$.

6. $\log_{25} 5 = 2$

Properties of logs:

If a, b, c, m and n are positive numbers with b and c not equal to zero, then:

1. Change of base

$$\log_b a = \frac{\log_c a}{\log_c b} = \frac{\ln a}{\ln b}$$

2. Power Property

$$\log_b m^n = n \log_b m$$

3. Product Property

$$\log_b mn = \log_b m + \log_b n$$

4. Quotient Property

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Example Solve for the value of x in each equation, explain.

a) $\ln 8 = \ln 2 + \ln x$

$e^{\ln 8} = e^{\ln 2x}$

$8 = 2x$

$4 = x$

b) $\ln 2 = \ln 14 - \ln x$

$e^{\ln 2} = e^{\ln \frac{14}{x}}$

$\frac{2}{1} = \frac{14}{x}$

$2x = 14$

$x = 7$

c) $\log 36 = 2 \log x$

$\log 36 = \log x^2$

$36 = x^2$

$6 = x$

ERROR ANALYSIS *Describe* **and correct the error in expanding the logarithmic expression.**

31. $\log_2 5x = (\log_2 5)(\log_2 x)$ ✗

32. $\ln 8x^3 = 3 \ln 8 + \ln x$ ✗

Describe: _____

Correct:

Example Expand the expression using the properties of logs.

$$\ln \frac{5\sqrt{x}}{y}$$

$\sqrt{x} = x^{1/2}$

$\ln 5 + \ln \sqrt{x} - \ln y$

$\ln 5 + \frac{1}{2} \ln x - \ln y$

$$\log_3 \left(\frac{x}{5y}\right)^2 = \log_3 \frac{x^2}{25y^2}$$

$\log_3 x^2 - (\log_3 25 + \log_3 y^2)$

$2\log_3 x - \log_3 25 - 2\log_3 y$

Example Expand the expression using the properties of logs.

$\log_{0.5} \dfrac{(x-1)^2}{\sqrt{3y}}$

$\log_{1/2} (x-1)^2 - [\log_{1/2} \sqrt{3y}]$

$2\log_{1/2}(x-1) - [\log_{1/2}\sqrt{3} + \log_{1/2}\sqrt{y}]$

$2\log_{1/2}(x-1) - [\frac{1}{2}\log_{1/2} 3 + \frac{1}{2}\log_{1/2} y]$

$2\log_{1/2}(x-1) - \frac{1}{2}\log_{1/2} 3 - \frac{1}{2}\log_{1/2} y$

$\ln \sqrt[5]{\dfrac{3x^2}{y^3}} = \ln \left(\dfrac{3x^2}{y^3}\right)^{1/5}$

$\frac{1}{5} \ln \frac{3x^2}{y^3}$

$\frac{1}{5}[\ln 3 + \ln x^2 - \ln y^3]$

$\frac{1}{5}[\ln 3 + 2\ln x - 3\ln y]$

$\frac{1}{5}\ln 3 + \frac{2}{5}\ln x - \frac{3}{5}\ln y$

Example
Condense the expression using the properties of logs.

$3 \ln y + 0.5 \ln (x+3)$

$\ln y^3 + \ln (x+3)^{1/2}$

$\ln y^3 \sqrt{x+3}$

$\dfrac{3}{5}\log_2 x - 3\log_2 (y-5)$

$\dfrac{\log_2 x^{3/5}}{\log_2 (y-5)^3}$

$\log_2 \left(\dfrac{\sqrt[5]{x}}{y-5}\right)^3$