

Objective: *Given a rational function students will be able to sketch its graph with appropriate vertical and horizontal asymptote².*

Def of Hole-It represents the fact that the function approaches the point, but is not actually defined on that precise X value. Not defined because not in the domain of the function. It makes denominator equal to zero, which is undefined.

- It possible you need to factor a quadratic expression. If we determine that the numerator and denominator share a common linear factor, it should be crossed out and the function simplified. There is a hole in the graph at the X-value where the shared linear factor equals zero. If

Note: while the graph of a rational function will never cross a vertical asymptote, **the graph may or may not cross a horizontal or slant asymptote**. Also, although the graph of a rational function may have many vertical asymptotes, the graph will have at most one horizontal (or slant) asymptote.

- A slant (oblique) asymptote occurs when the polynomial in the numerator is a higher degree than the polynomial in the denominator. To find the slant asymptote you must divide the numerator by the denominator using either long division or synthetic division.

Example 2: Sketch the graph of the rational Function

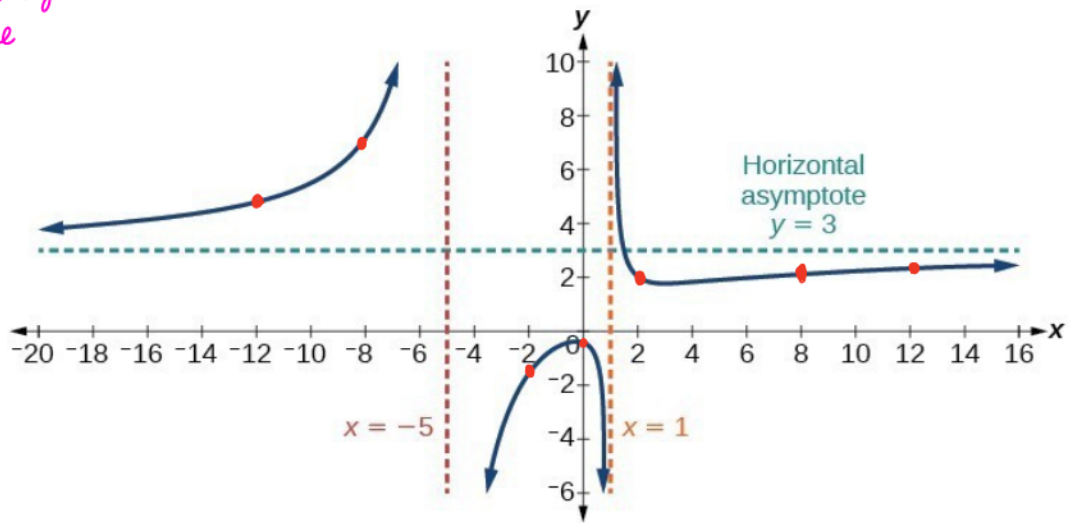
$$f(x) = \frac{3x^2 + 2}{x^2 + 4x - 5} = \frac{3x^2 + 2}{(x+5)(x-1)}$$

V.A : $x = -5, x = 1$

H.A : $y = 3$

X-int ($y=0$) | y-int ($x=0$)
 $0 = 3x^2 + 2$ | $y = -\frac{2}{5}$
 $-\frac{2}{3} = \frac{3x^2}{3}$ | $(0, -\frac{2}{5})$
 $-\frac{2}{3} = x^2$
 $i\sqrt{\frac{2}{3}} = \text{imagi}$
 none

x	y
-12	4.77
-8	7.19
-2	-1.56
0	-.4
2	2
8	2.15
12	2.32



Example 3: Sketch the graph of the rational Function

$$f(x) = \frac{x^2 - x - 12}{2x^2 - 4x - 16} = \frac{(x-4)(x+3)}{2(x^2 - 2x - 8)} = \frac{(x-4)(x+3)}{2(x-4)(x+2)} = \frac{x+3}{2(x+2)}$$

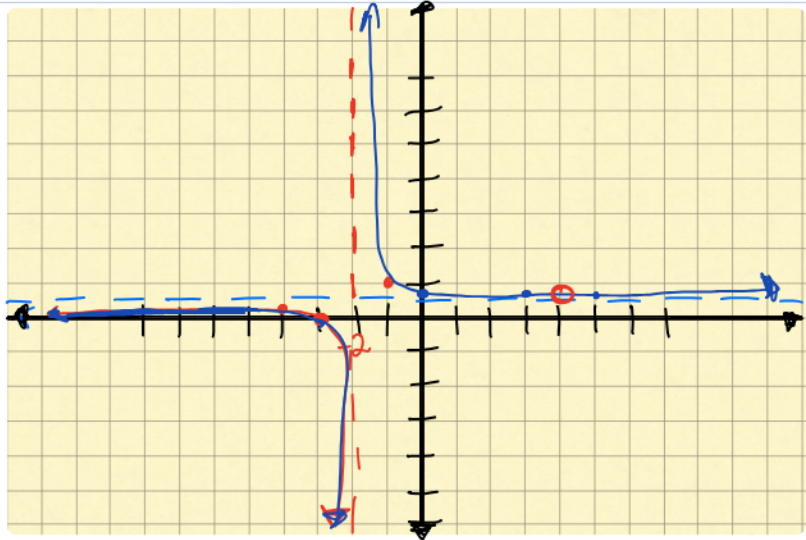
Vertical asy = $x = -2$

Horizontal asy $y = 1/2$

Hole: $(4, 7/2)$

$$f(4) = \frac{4+3}{2(4+2)} = \frac{7}{12}$$

X-int ($y=0$)	Y-int ($x=0$)
$0 = x+3$	$y = \frac{3}{2(2)} = 3/4$
$-3 = 0$	$(0, 3/4)$
$(-3, 0)$	



x	y
-4	.25
-1	1
3	.6
5	.57