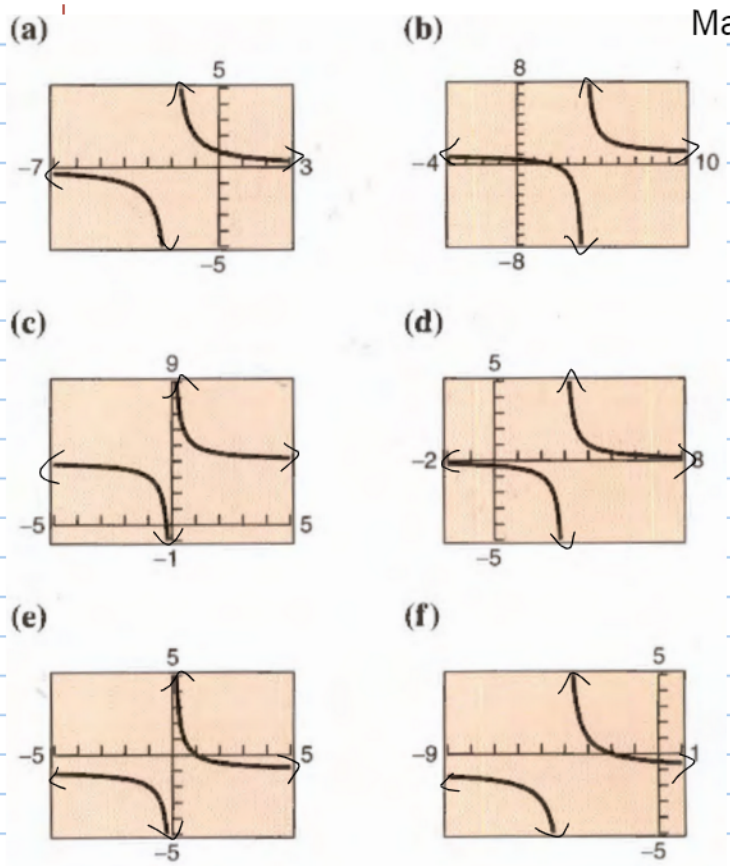
Section 2.6 Rational Functions and Asymptotes

Objective Given a rational function students will be able to find vertical, horizontal or slant asymptotes, find x and y intercepts, find domain and use graphing calculator to verify results.

Part 1: Study Problems Page 195 # 7- 15,21, 27, 54

Part 2: Study Problems Page 195#16, 18, 29, 40-43

Match the graph with its functions



7. $f(x) = \dfrac{2}{x+2}$

9. $f(x) = \dfrac{4x+1}{x}$

11. $f(x) = \dfrac{x-2}{x-4}$

8. $f(x) = \dfrac{1}{x-3}$

10. $f(x) = \dfrac{1-x}{x}$

12. $f(x) = -\dfrac{x+2}{x+4}$

KEY CONCEPT *For Your Notebook*

Graphs of Rational Functions

Let $p(x)$ and $q(x)$ be polynomials with no common factors other than ± 1. The graph of the following rational function has the characteristics listed below.

$$f(x) = \frac{p(x)}{q(x)} \rightarrow \frac{\text{m degrees}}{\text{n degrees}}$$

1. The x-intercepts of the graph of f are the real zeros of $p(x)$. Numerator = 0
2. The graph of f has a vertical asymptote at each real zero of $q(x)$. Denominator = 0
3. The graph of f has at most one horizontal asymptote, which is determined by the degrees m and n of $p(x)$ and $q(x)$.

$m < n$	The line $y = 0$ is a horizontal asymptote. deg. num. < deg. deno.
$m = n$	The line $y = \frac{a_m}{b_n}$ is a horizontal asymptote.
$m > n$	The graph has no horizontal asymptote. slant asymptote The graph's end behavior is the same as the graph of $y = \frac{a_m}{b_n}x^{m-n}$.

Identify the rational function horizontal asymptote:

Deg of numerator < Deg of denominator: y = 0	Deg of numerator = Deg of denominator: y = a/b	Deg of numerator > Deg of denominator: slant
$y = \frac{6}{x^2+1}$	$y = \frac{2x^2}{x^2-9}$	$y = \frac{x^2-2x-3}{x-4}$
$y = \frac{4}{x^2+2}$	$y = \frac{2x+1}{x-3}$	$y = \frac{x^2+3x-4}{x-2}$
	$y = \frac{-4}{x+2} - 1$	$y = \frac{3x^2}{x-1}$

Example

$f(x) = \dfrac{3}{x^2+2}$

a) *find the domain*

b) *Identify the horizontal asymptote*

c) *Identify the vertical asymptote*

d) *x and y intercepts*

Justify your response

use a graphing calculator to verify your results

a) $x^2+2 \neq 0$

$x^2 \neq -2$

$x \neq \pm\sqrt{-2}$

↑ imaginary

$\{x \in \mathbb{R}\}$

b) horiz. asy. (Deg num < Deg. Den)

$y = 0$

c) Vertical asy

none, b/c deno. has NO real zeros (look @ part a)

d) x-int (y=0)

$(x^2+2)\, 0 = \dfrac{3}{x^2+2} \cdot (x^2+2)$

$0 \neq 3$

no x-int

y-int (x=0

$y = \dfrac{3}{0^2+2} = 3/2$ $(0, 3/2)$